Noran Engineering, Inc.
Nastran Finite Element Analysis Software

Composite Analysis for Maritime Application

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The paper reviews the various methods used to represent composite structures in Finite Element Analysis at both global and local levels and attempts to give guidelines on applicability.

Two Dimensional Plane Stress Elements

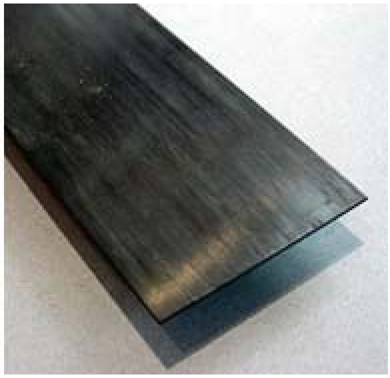
Three Dimensional Solid Elements

Two Dimensional Plane Strain Elements

Material Definitions

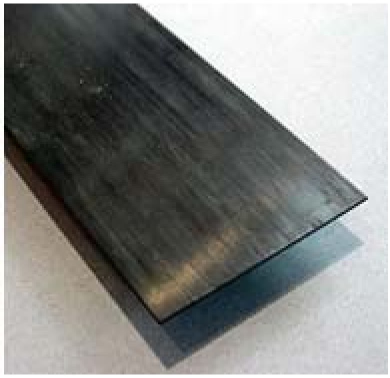
ISOTROPIC - the same material properties in all directions, steel is a typical example.

ANISOTROPIC - different material properties in all directions, a chunk of volcanic rock is an example.



ORTHOTROPIC – special case of anisotropic , clear material directionality in 3 directions –represents a carbon fiber/resin matrix for example, where the along axis, transverse axis and through thickness axis are different.

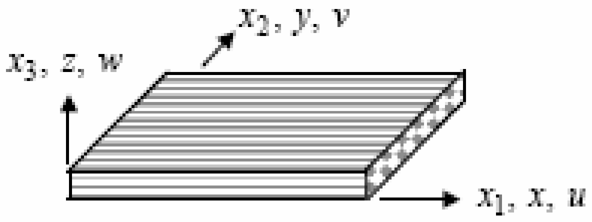
Material Definitions



2D ORTHOTROPIC - A further simplification where we ignore the through thickness variation.

This is the usual starting point for what we call Classical laminate Theory

- **Basis for simple ply calculation tools**
- **The foundation of most FE solutions**

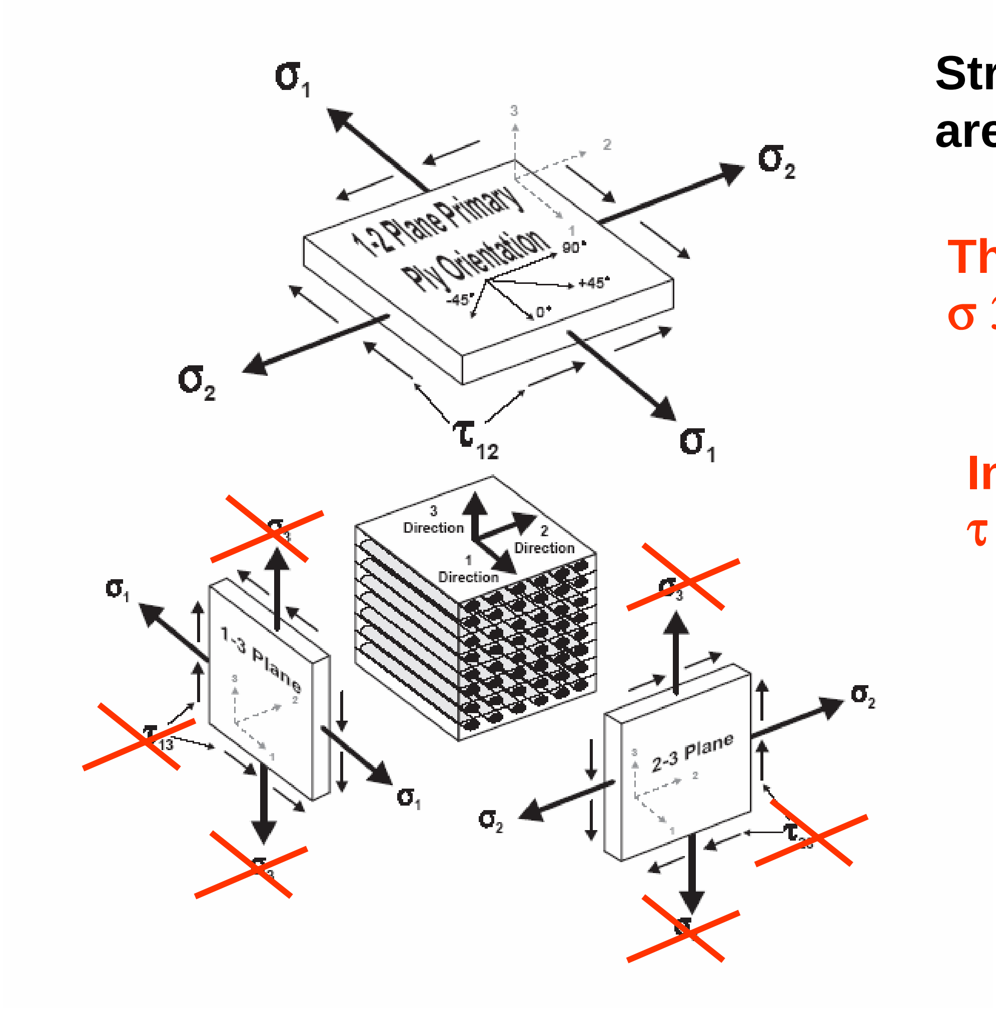


Plane Stress - Thin bodies

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

$$\therefore \varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$$

Material Definitions

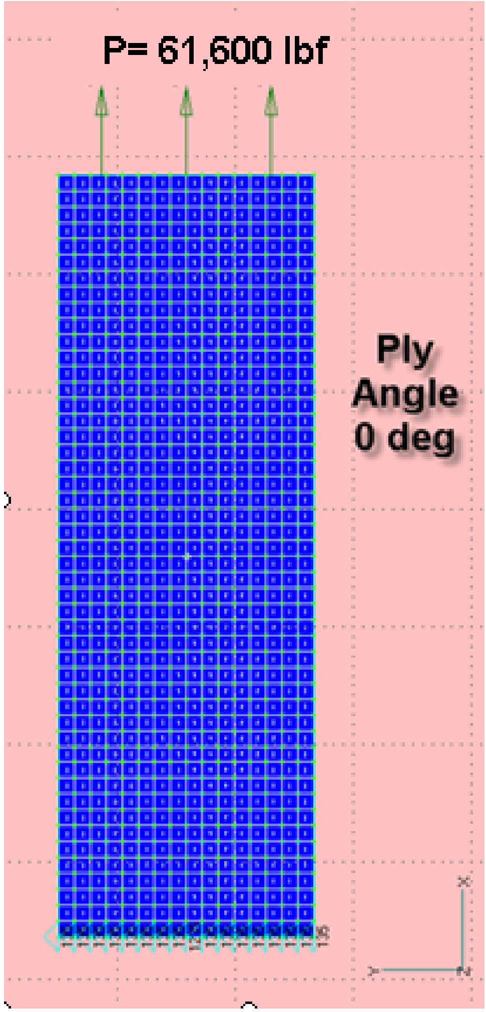


Stresses that are ignored:

Through Thickness
σ 33

InterLaminar
τ 23 τ13

Ply Behaviour



Single Ply

Material Allowables

xt	154,000	psi
xc	88,500	psi
yt	4,500	psi
yc	17,100	psi
s	10,400	psi

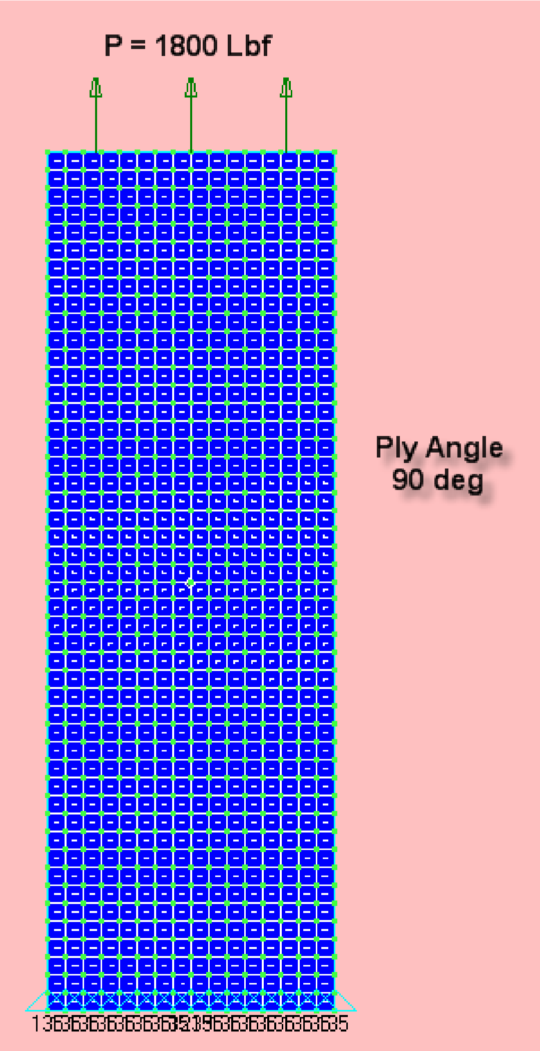
Stress state

s 1 = 154,000 psi

s 2 = 0

t 12 = 0

Ply Behaviour



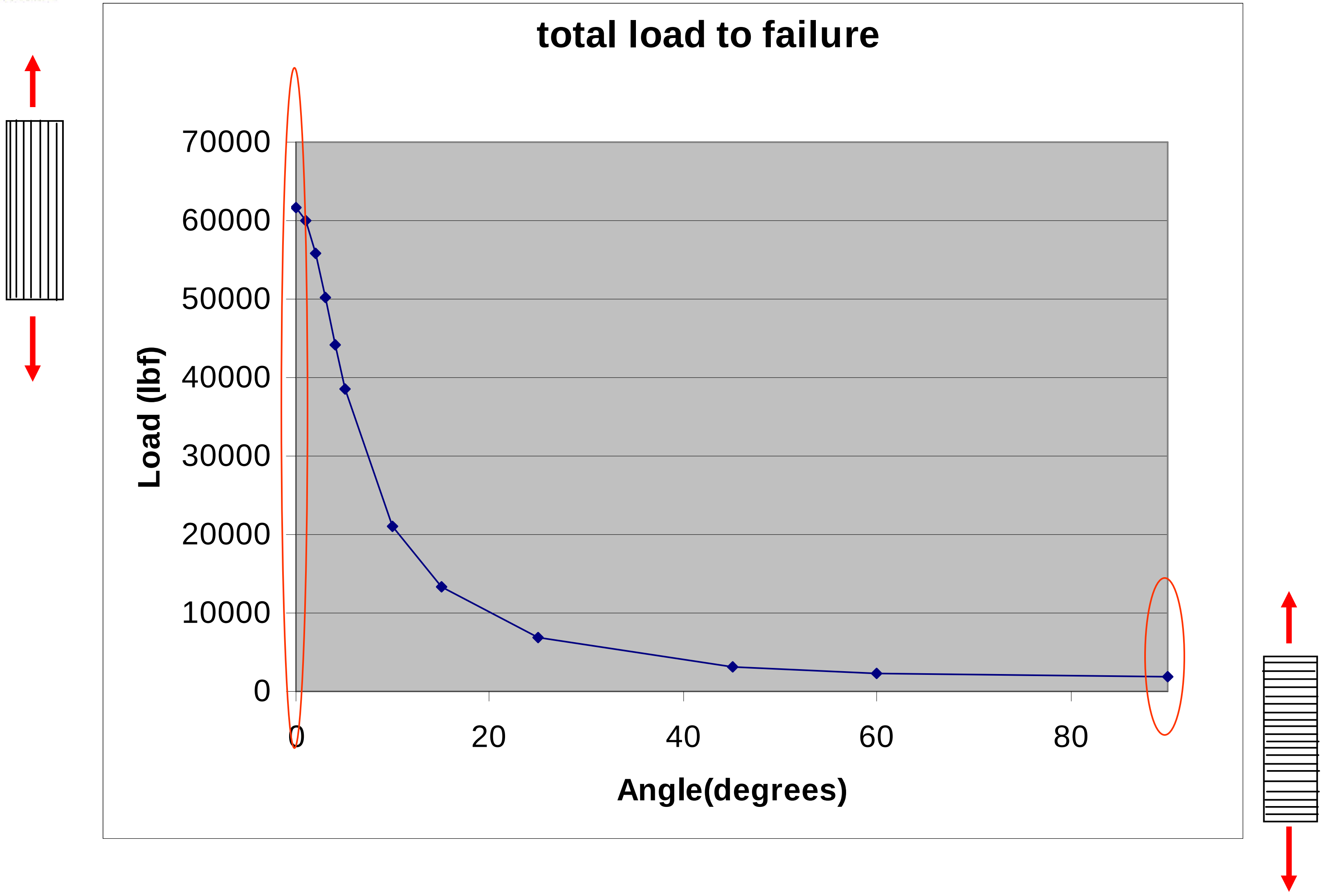
Single Ply

Material Allowables

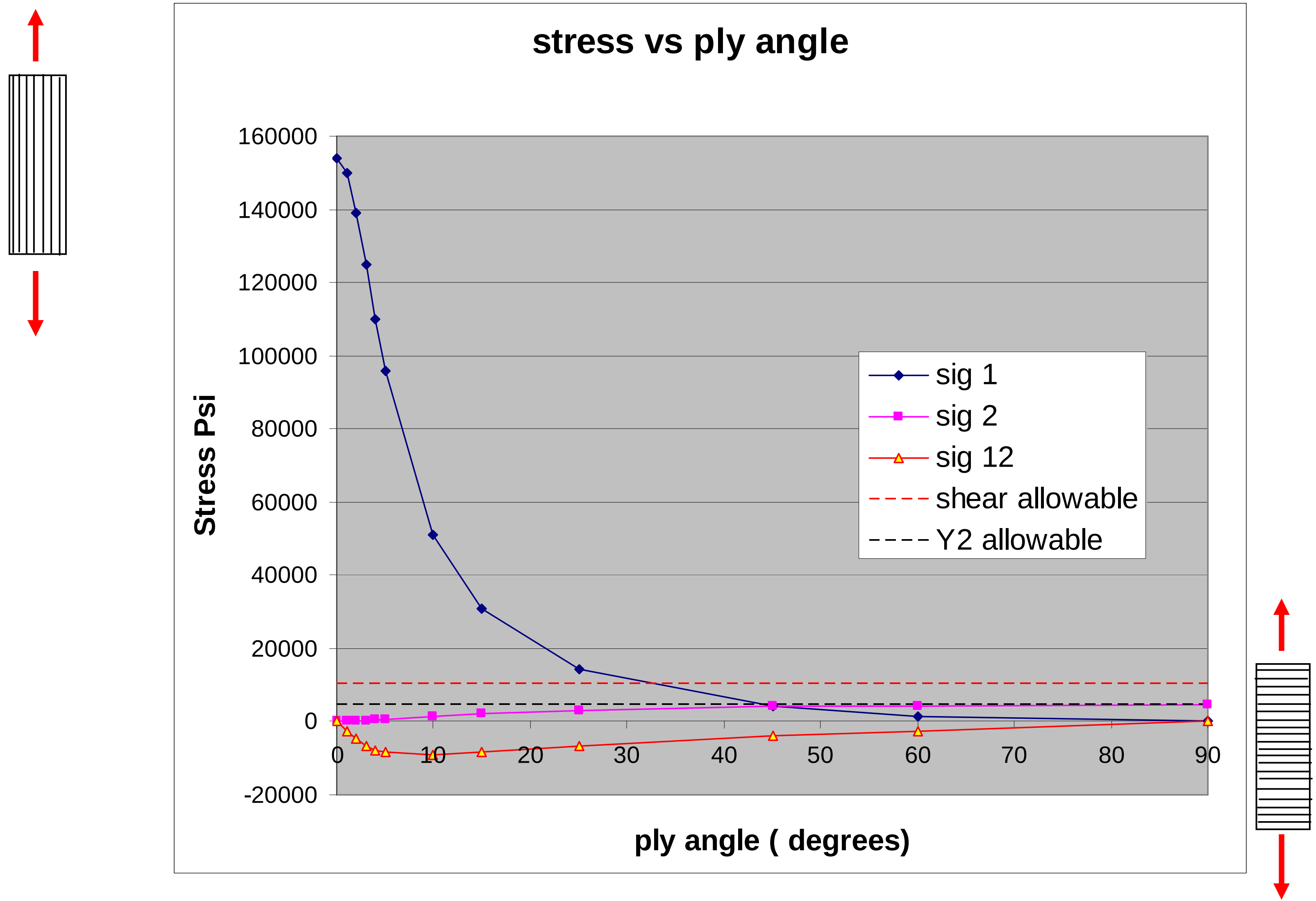
xt	**154,000**	psi
xc	88,500	psi
yt	**4,500**	psi
yc	17,100	psi
s	10,400	psi

Stress state
s 1 = 0
s 2 = 4,500 psi
t 12 = 0

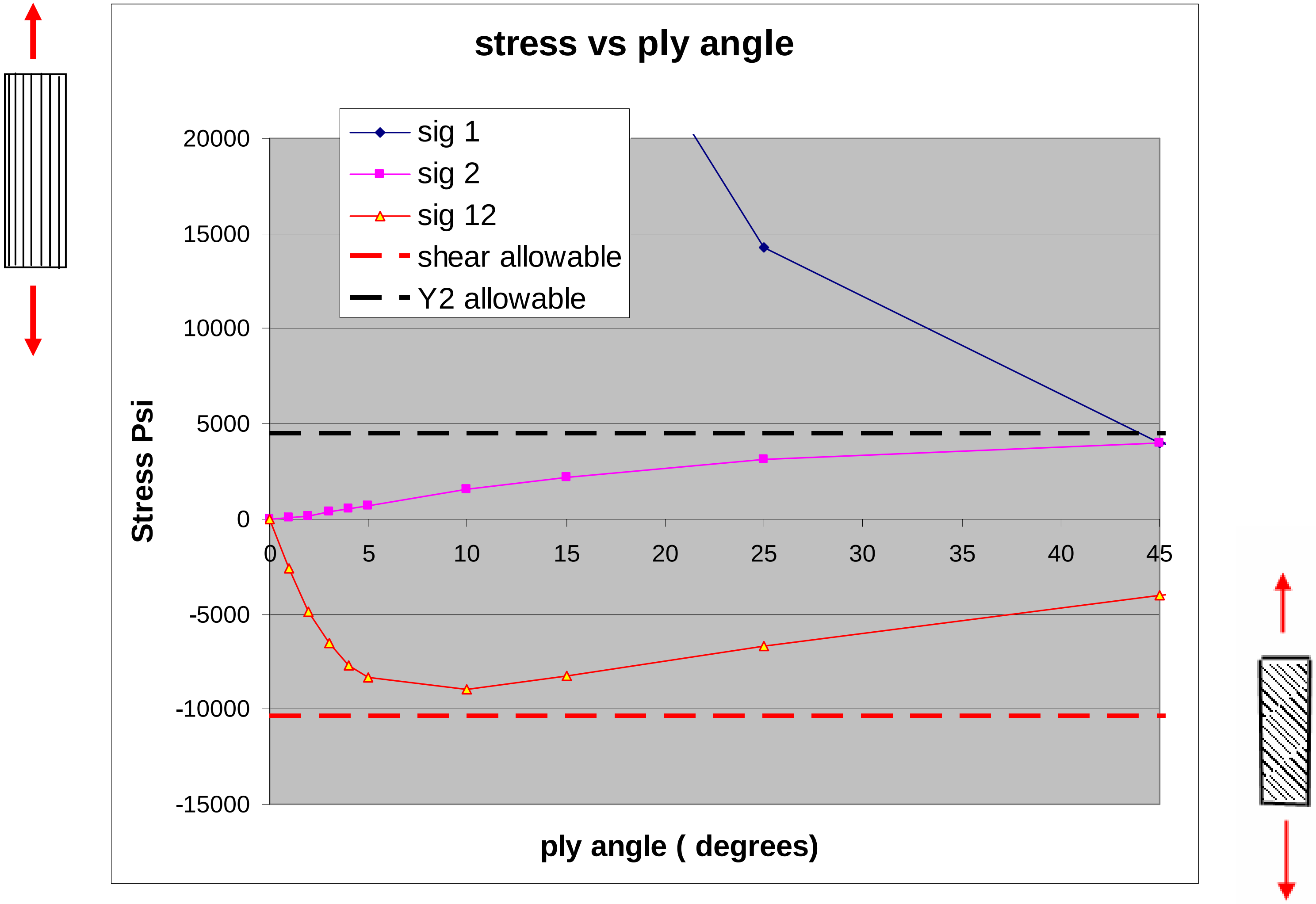
Ply Behaviour



Material Behaviour



Material Behaviour



Material Behaviour

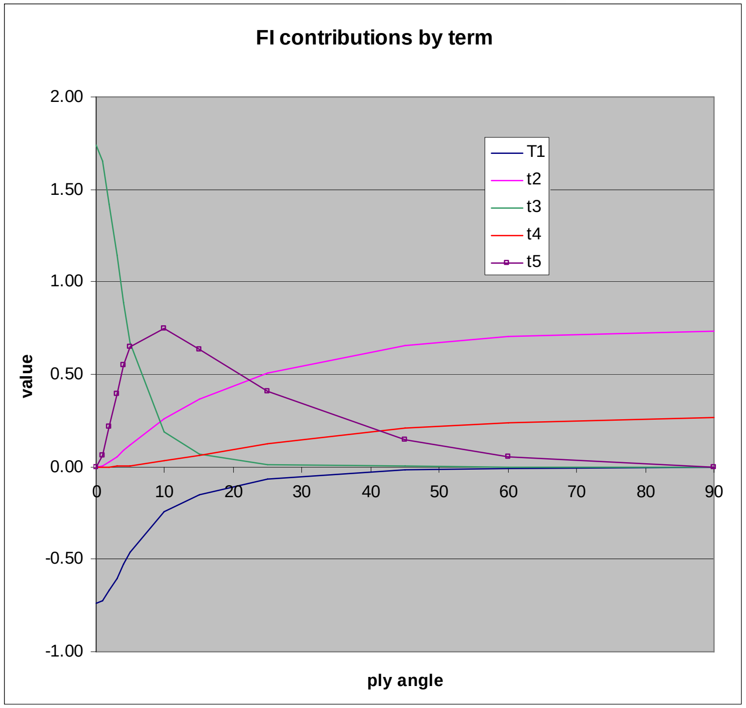
How do we predict the intermediate ply angle strengths?

A **failure theory** analogous to Von Mises stresses for Isotropic materials is used to predict failure.

Tsai-Wu typical model

$$\left(\frac{1}{X_t}-\frac{1}{X_c}\right)\sigma_1+\left(\frac{1}{Y_t}-\frac{1}{Y_c}\right)\sigma_2+\frac{\sigma_1^2}{X_tX_c}+\frac{\sigma_2^2}{Y_tY_c}+\frac{\tau_{12}^2}{S^2}+2F_{12}\sigma_1\sigma_2=\text{F.I.}$$

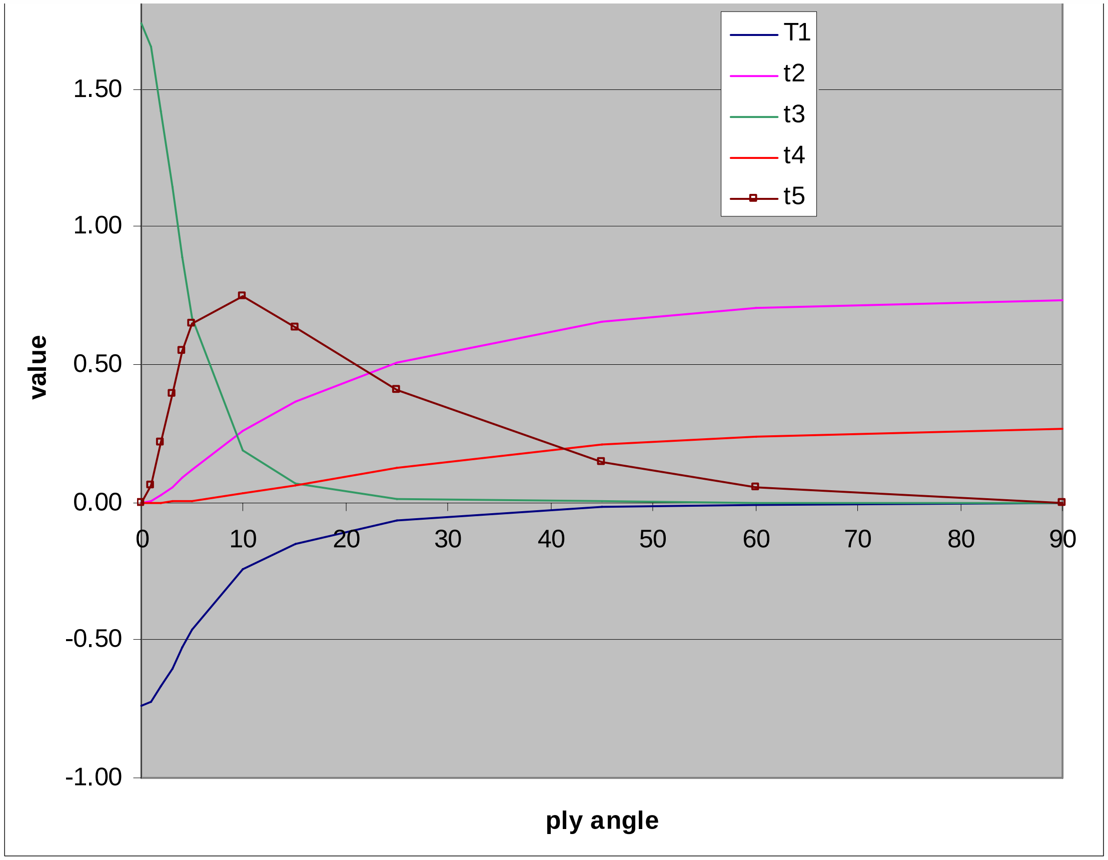
Xt	**tension limit, along fiber**
Xc	**compression limit, along fiber**
Yt	**tension limit, transverse fiber**
Yc	**compression limit, transverse fiber**
S	**shear limit**
F12	**interaction term**



Failure Index > 1.0 is bad news

Failure Theories

$$\left(\frac{1}{X_t}-\frac{1}{X_c}\right)\sigma_1+\left(\frac{1}{Y_t}-\frac{1}{Y_c}\right)\sigma_2+\frac{\sigma_1^2}{X_tX_c}+\frac{\sigma_2^2}{Y_tY_c}+\frac{\tau_{12}^2}{S^2}+2F_{12}\sigma_1\sigma_2=\text{F.I.}$$



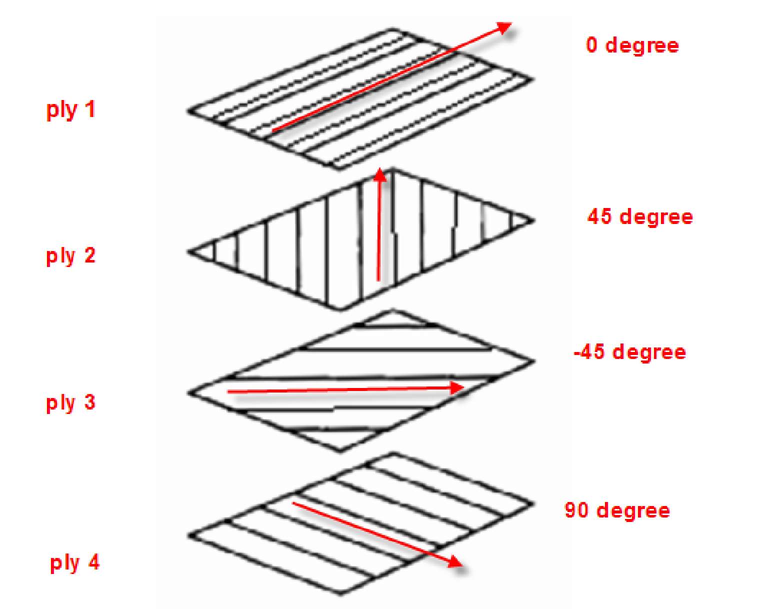
What about F12?

Available Failure Theories

Theory	Failure Index	Remarks
Hill	$\frac{\sigma_1^2}{x^2} - \frac{\sigma_1\sigma_2}{x^2} + \frac{\sigma_2^2}{y^2} + \frac{\tau_{12}^2}{s^2} = \text{F.I.}$	Orthotropic materials with equal strengths in tension and compression.
Hoffman	$\left(\frac{1}{x_t} - \frac{1}{x_c}\right)\sigma_1 + \left(\frac{1}{y_t} - \frac{1}{y_c}\right)\sigma_2 + \frac{\sigma_1^2}{x_t x_c} + \frac{\sigma_2^2}{y_t y_c} + \frac{\tau_{12}^2}{s^2} - \frac{\sigma_1\sigma_2}{x_t x_c} = \text{F.I.}$	Orthotropic materials under a general state of plane stress with unequal tensile and compressive strengths.
Tsai-Wu	$\left(\frac{1}{x_t} - \frac{1}{x_c}\right)\sigma_1 + \left(\frac{1}{y_t} - \frac{1}{y_c}\right)\sigma_2 + \frac{\sigma_1^2}{x_t x_c} + \frac{\sigma_2^2}{y_t y_c} + \frac{\tau_{12}^2}{s^2} + 2F_{12}\sigma_1\sigma_2 = \text{F.I.}$	Orthotropic materials under a general state of plane stress with unequal tensile and compressive strengths.
LaRC02	***Dávila, C.G., Jaunky, N., and Goswami, S., Failure Criteria for FRP Laminates in Plane Stress, , 2003***	**Unidirectional materials in a general state of plane stress with unequal tensile and compressive strengths**
Max Stress	$\text{Max}\left[\left(\frac{\sigma_1}{X_t}\right), \left(\frac{\sigma_2}{Y_t}\right), \left(\frac{\lvert\tau_{12}\rvert}{S}\right)\right]$	None
Max Strain	$\text{Max}\left[\left(\frac{\varepsilon_1}{X_t}\right), \left(\frac{\varepsilon_2}{Y_t}\right), \left(\frac{\lvert\gamma_{12}\rvert}{S}\right)\right]$	None
Puck	***A. Puck and H. Schürmann: "Failure analysis of FRP laminates by means of physically based phenomenological models", 1998***	**Unidirectional materials in a general state of plane stress with unequal tensile and compressive strengths**

Ply Layups

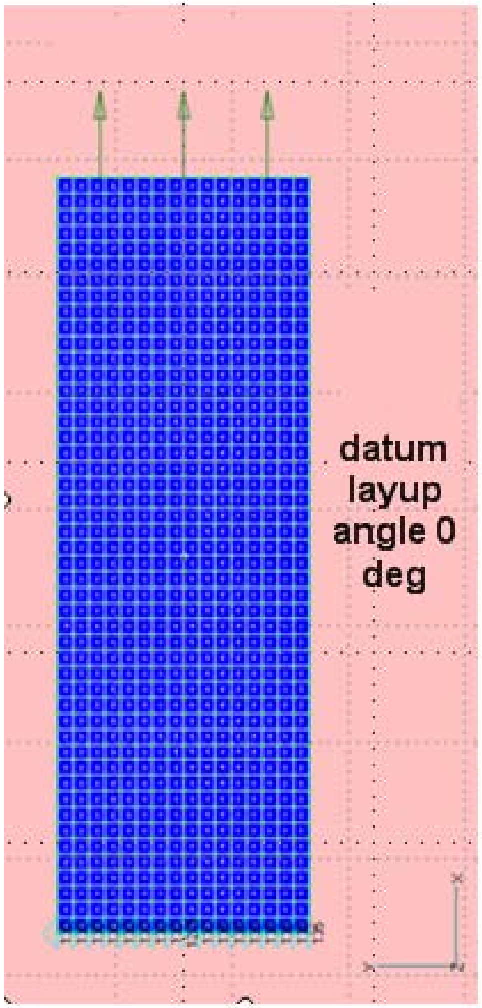
- Single Ply directions exposes directional weakness
- Ply layups used of multiple orientation to improve this



A laminate made up of 4 plies

- Shorthand 0/45/-45/90
- Tuning the layup orientation, thickness and stacking order is key to optimum design

Stackup Behaviour



Ply Layup

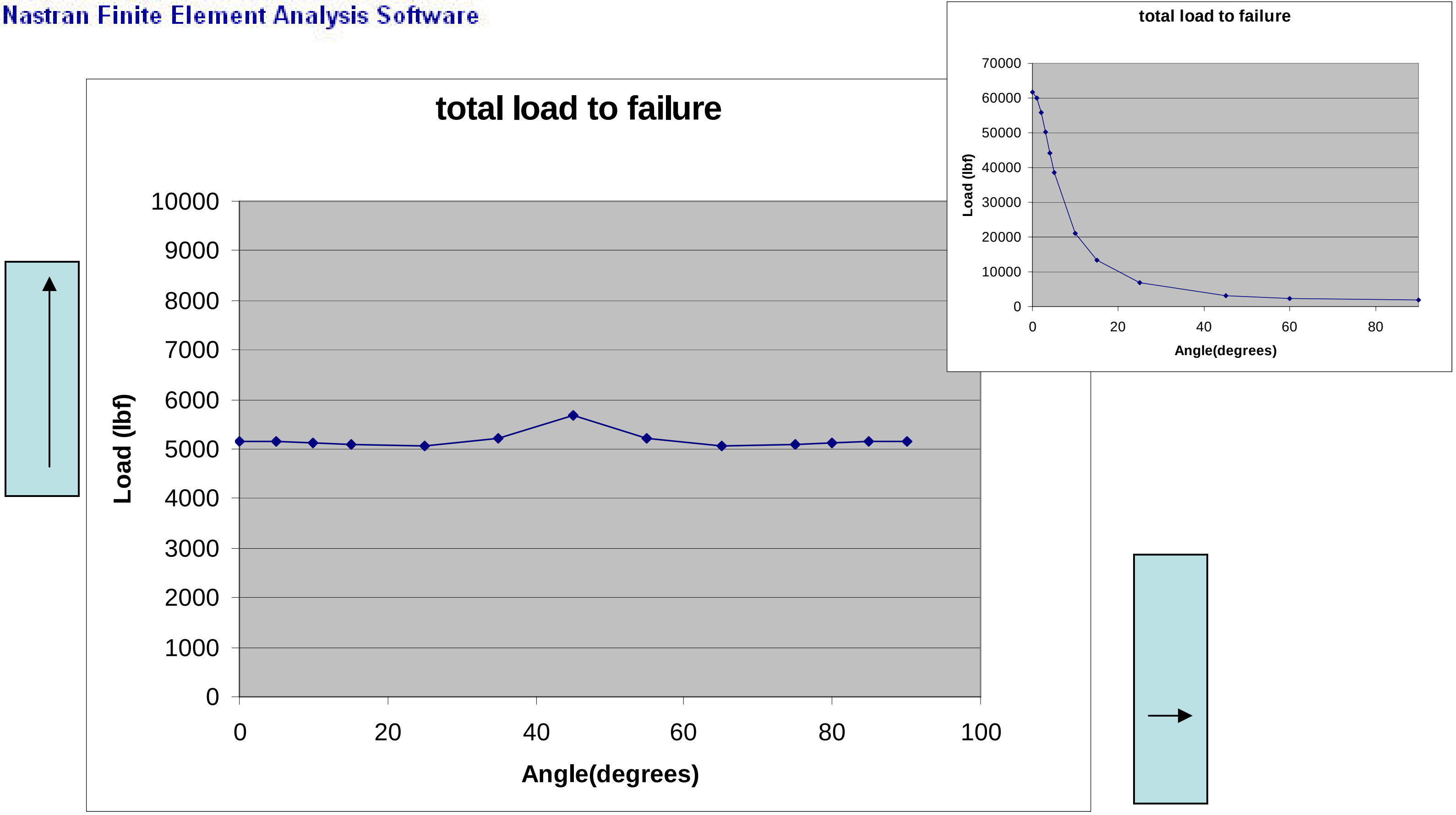
Material Allowables

xt	154,000	psi
xc	88,500	psi
yt	4,500	psi
yc	17,100	psi
s	10,400	psi

0
90
-45
45
45
-45
90
0

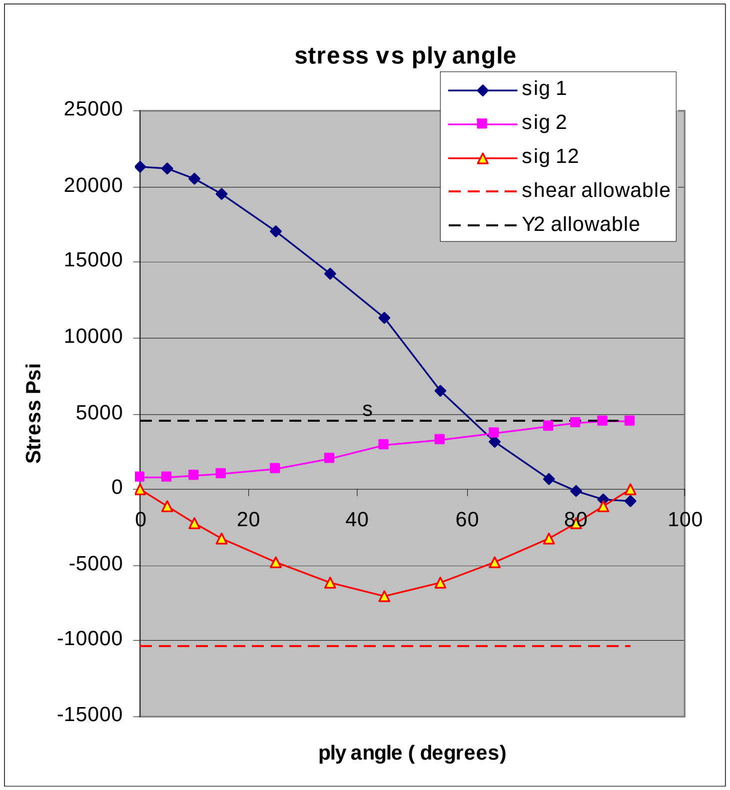
All : t/8

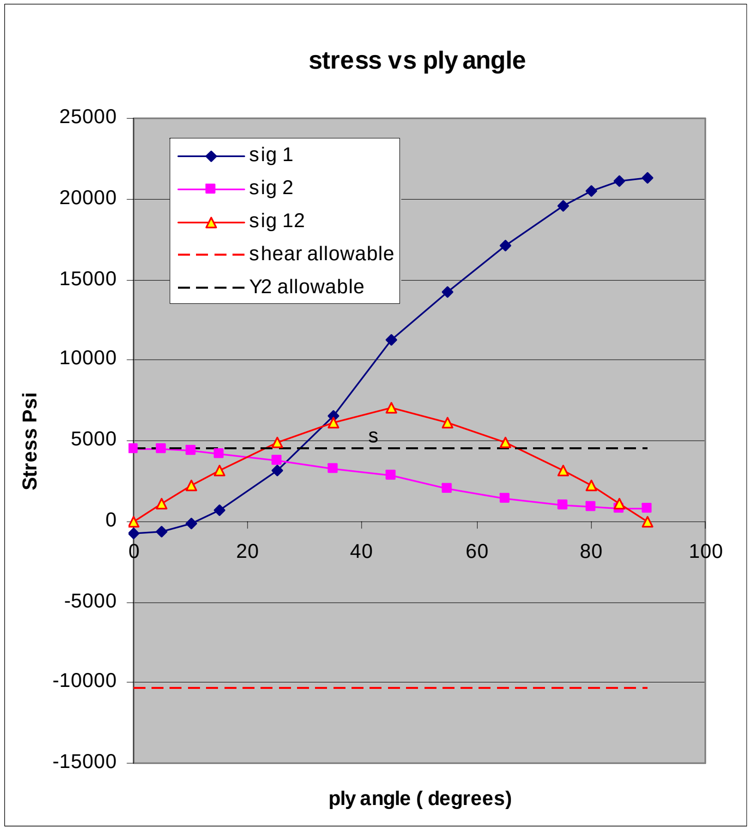
Ply Layups



- Previous Single Ply replaced by 0/90/-45/45/45/-45/90/0
- Maximum Strength is reduced, but now very predictable
- No Optimization! Sometimes called ‘black’ isotropic material ….

Ply Layups

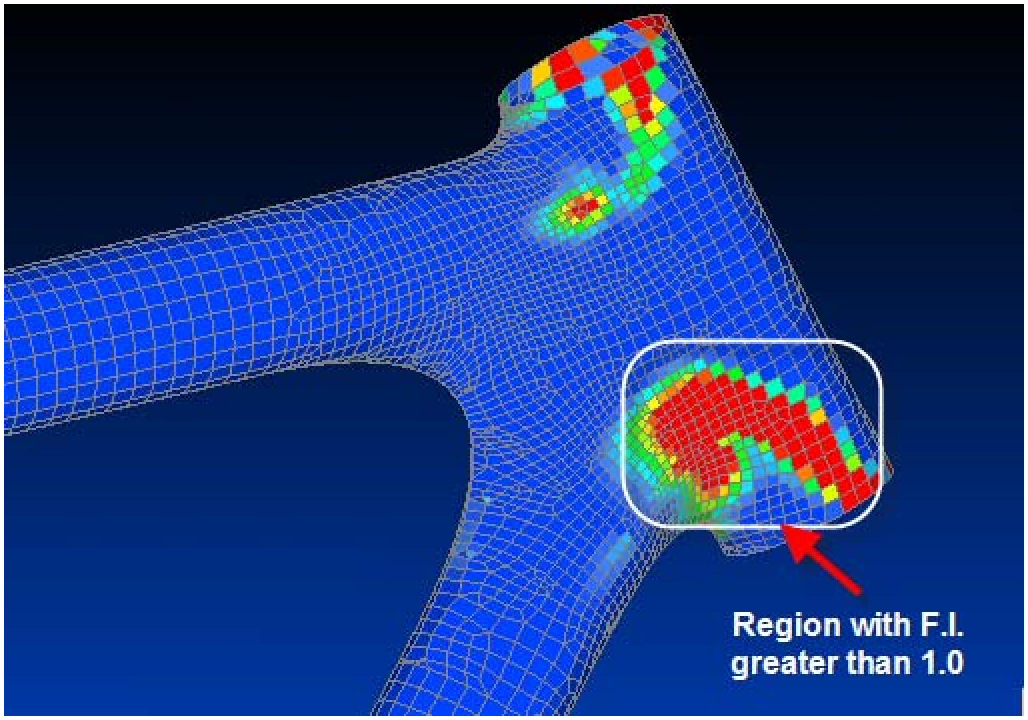




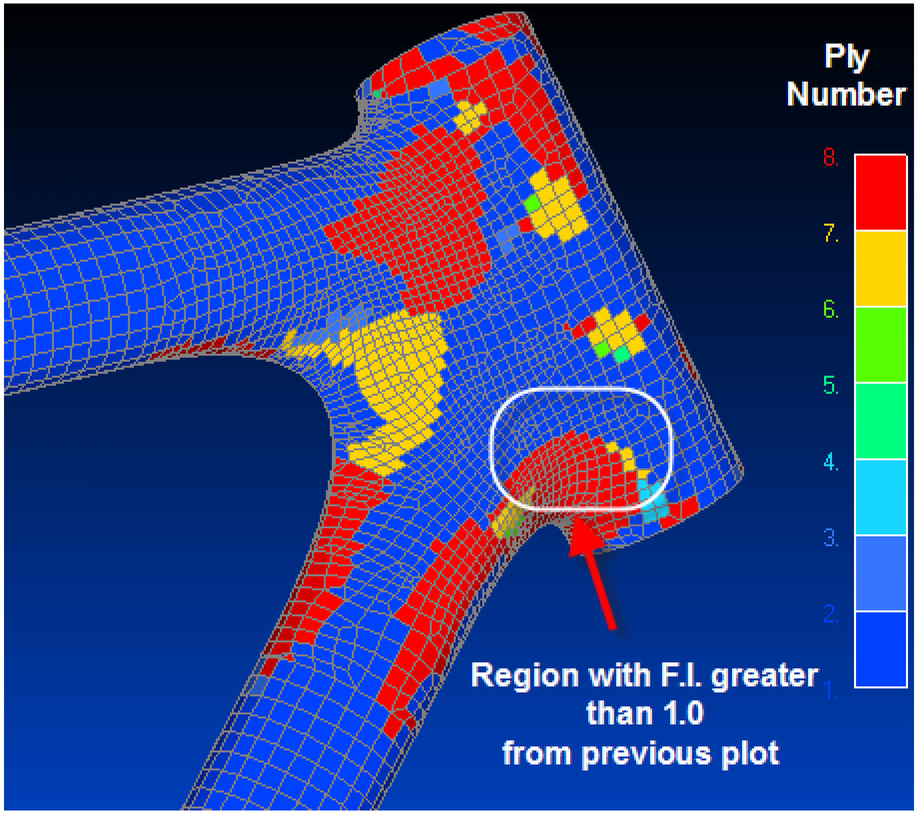
Top: 0 degree ply

Second: 90 degree ply

→ 0
→ 90
-45
45
45
-45
90
0

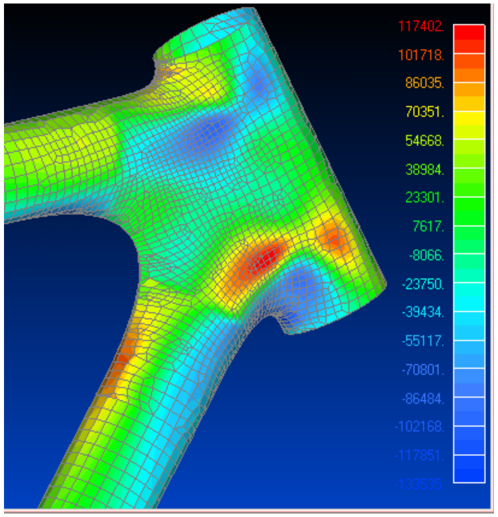


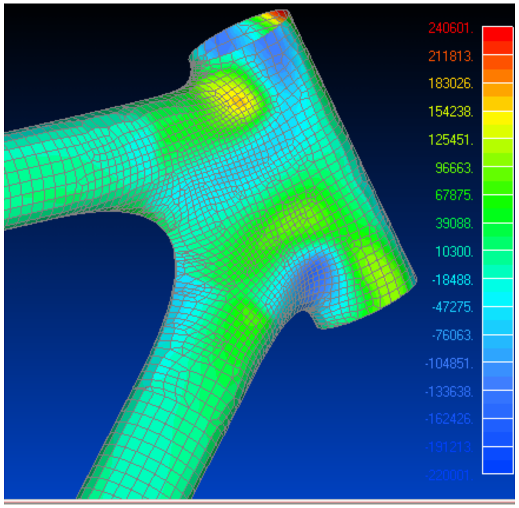
- Identify regions where F.I. shows failure in the layup

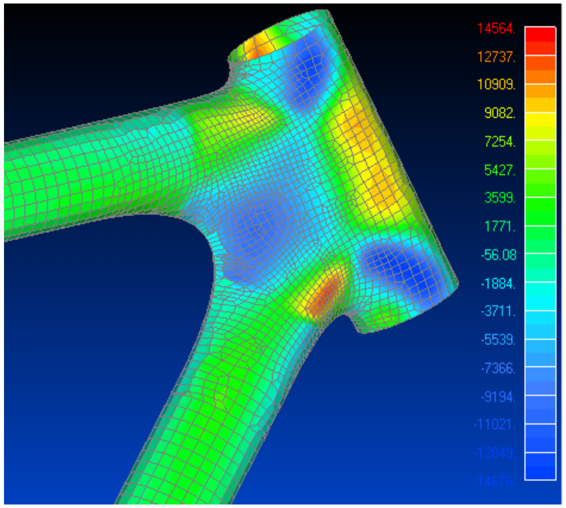


Region with F.I. greater than 1.0 from previous plot

- Identify which plies are failing in the layup in that region



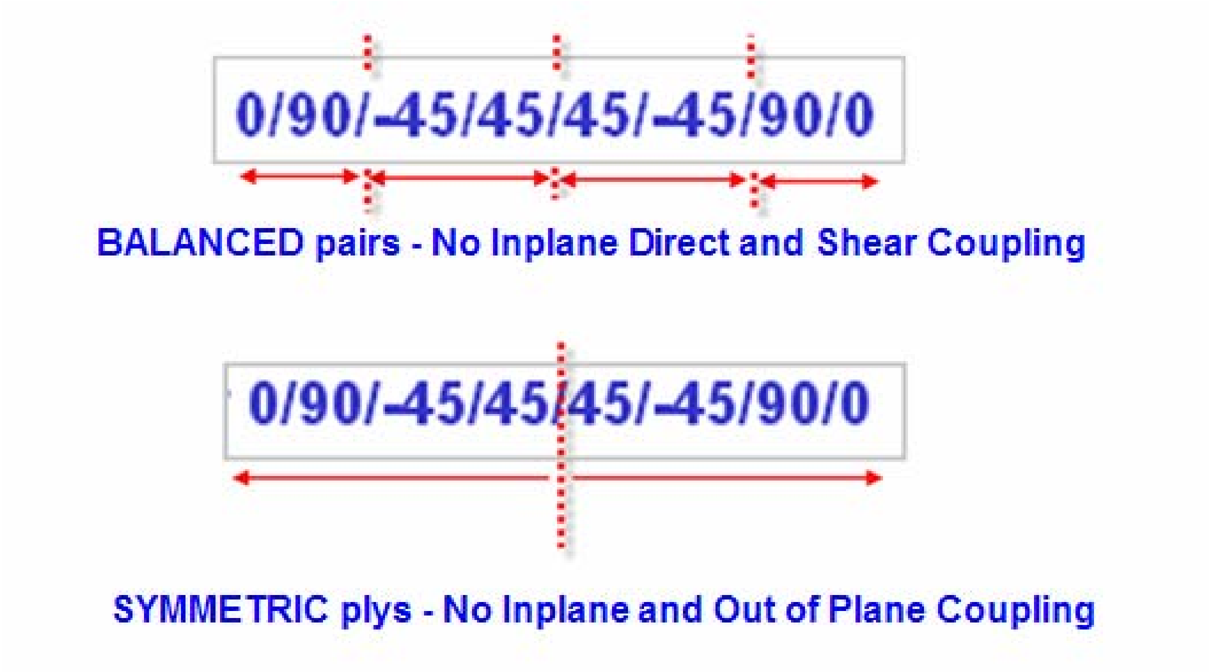




- **Review Direct X, Direct Y and Shear XY ply stresses in the individual ply**
- **Assess major mode of failure**
- **Assess coupling through plies**
- **Redesign if required**

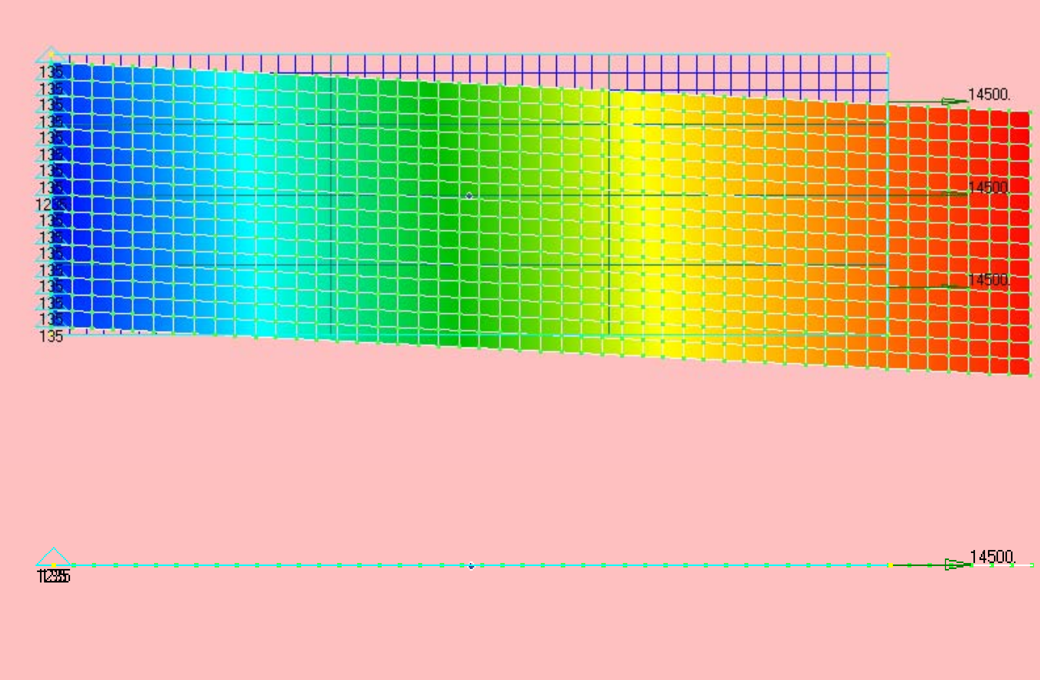
Ply Layups

- why 0/90/-45/45/45/-45/90/0 choice?

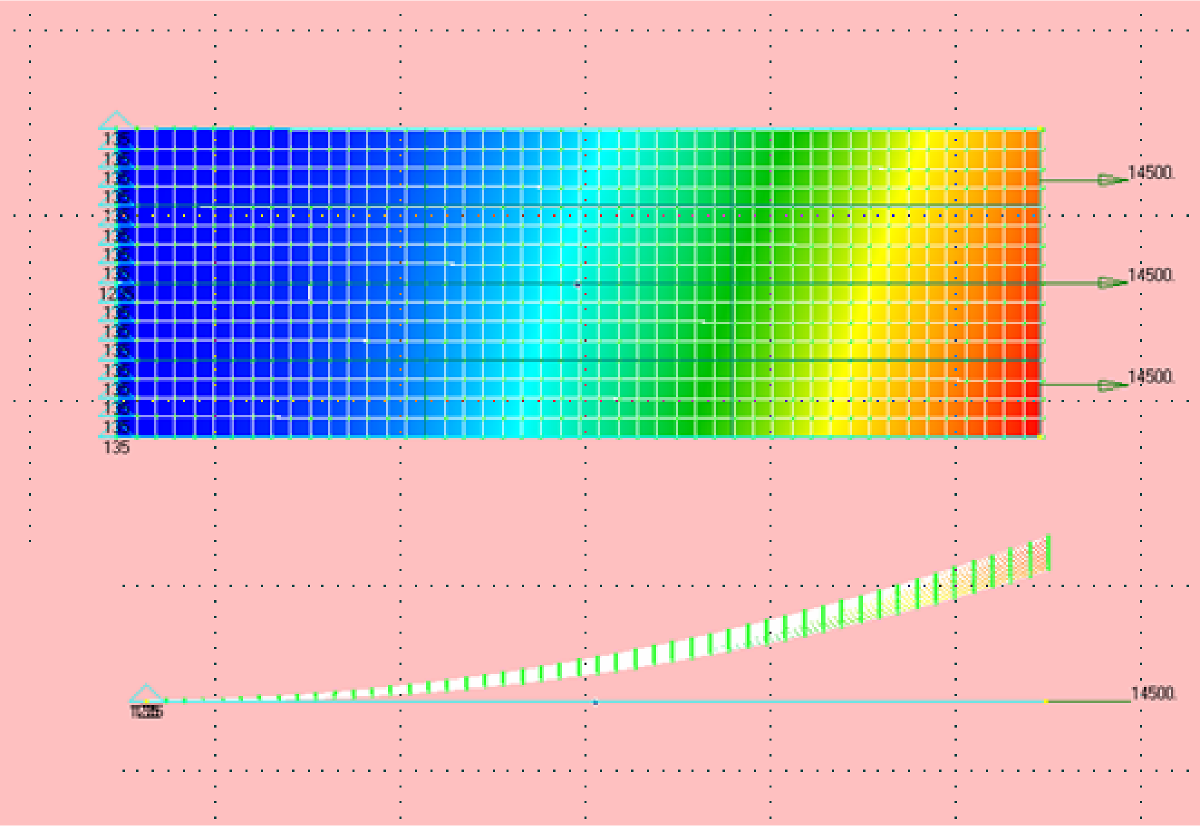


BALANCED pairs - No Inplane Direct and Shear Coupling

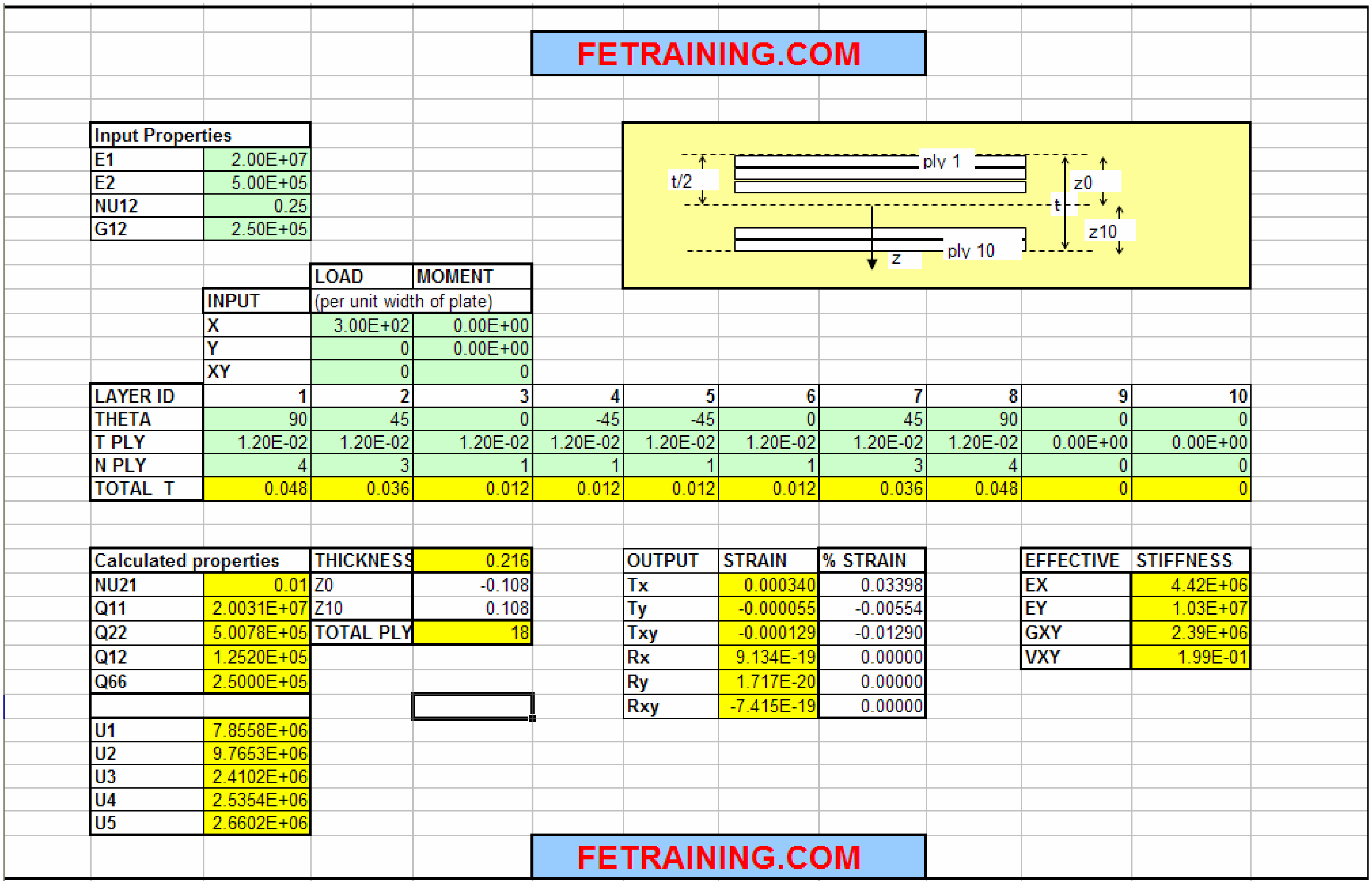
SYMMETRIC plys - No Inplane and Out of Plane Coupling



Unbalanced but Symmetric



Balanced but Non Symmetric



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Input Properties	
E1	2.00E+07
E2	5.00E+05
NU12	0.25
G12	2.50E+05

INPUT	LOAD	MOMENT
	(per unit width of plate)	
X	3.00E+02	0.00E+00
Y	0	0.00E+00
XY	0	0

LAYER ID	1	2	3	4	5	6	7	8	9	10
THETA	90	45	0	-45	-45	0	45	90	0	0
T PLY	1.20E-02	1.20E-02	1.20E-02	1.20E-02	1.20E-02	1.20E-02	1.20E-02	1.20E-02	0.00E+00	0.00E+00
N PLY	4	3	1	1	1	1	3	4	0	0
TOTAL T	0.048	0.036	0.012	0.012	0.012	0.012	0.036	0.048	0	0

Calculated properties		THICKNESS	0.216
NU21	0.01	Z0	-0.108
Q11	2.0031E+07	Z10	0.108
Q22	5.0078E+05	TOTAL PLY	18
Q12	1.2520E+05		
Q66	2.5000E+05		
U1	7.8558E+06		
U2	9.7653E+06		
U3	2.4102E+06		
U4	2.5354E+06		
U5	2.6602E+06		

OUTPUT	STRAIN	% STRAIN
Tx	0.000340	0.03398
Ty	-0.000055	-0.00554
Txy	-0.000129	-0.01290
Rx	9.134E-19	0.00000
Ry	1.717E-20	0.00000
Rxy	-7.415E-19	0.00000

EFFECTIVE	STIFFNESS
EX	4.42E+06
EY	1.03E+07
GXY	2.39E+06
VXY	1.99E-01

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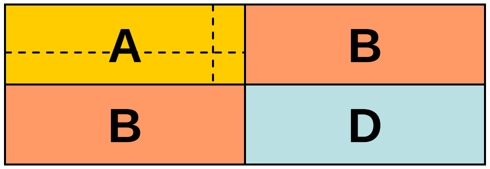


A	B
B	D

Bal + Sym

7.54E+05	2.43E+05	9.15E-14	-1.82E-12	1.00E-12	-6.79E-13
2.43E+05	7.54E+05	2.88E-11	1.00E-12	2.76E-12	-7.09E-13
9.15E-14	2.88E-11	2.55E+05	-6.79E-13	-7.09E-13	8.53E-13
-1.82E-12	1.00E-12	-6.79E-13	9.15E+02	5.37E+01	3.37E+01
1.00E-12	2.76E-12	-7.09E-13	5.37E+01	5.10E+02	3.37E+01
-6.79E-13	-7.09E-13	8.53E-13	3.37E+01	3.37E+01	6.29E+01

Bal

7.54E+05	2.43E+05	9.15E-14	2.81E+03	1.00E-12	-6.79E-13
2.43E+05	7.54E+05	2.88E-11	1.00E-12	-2.81E+03	-9.36E-13
9.15E-14	2.88E-11	2.55E+05	-6.79E-13	-9.36E-13	8.53E-13
2.81E+03	1.00E-12	-6.79E-13	7.12E+02	5.37E+01	3.37E+01
1.00E-12	-2.81E+03	-9.36E-13	5.37E+01	7.12E+02	3.37E+01
-6.79E-13	-9.36E-13	8.53E-13	3.37E+01	3.37E+01	6.29E+01

Sym

1.05E+06	4.90E+05	2.34E+05	-2.50E-12	-3.30E-12	3.07E-14
4.90E+05	2.46E+06	2.34E+05	-3.30E-12	-1.46E-11	1.16E-12
2.34E+05	2.34E+05	5.17E+05	3.07E-14	1.16E-12	-6.82E-13
-2.50E-12	-3.30E-12	3.07E-14	1.25E+03	7.60E+02	6.52E+02
-3.30E-12	-1.46E-11	1.16E-12	7.60E+02	1.47E+04	6.52E+02
3.07E-14	1.16E-12	-6.82E-13	6.52E+02	6.52E+02	8.65E+02

Two Dimensional composite thin shells

Earliest implementation of composites in FEA

- Satellite Structures
- Missile structures
- Very early military aircraft wings

Originally used smeared ‘pseudo’ orthotropic in isotropic material model

Then equivalent 2d orthotropic material using CLT – no ply definition in FEA!
Very painful process – spot post processing only using CLT programs

Decoupled bending, membrane and shear properties for Honeycomb – still sometimes used today

Two dimensional thin shells

Some general assumptions for thin shell behavior are:

The ratio of plate thickness to smallest bounding plate edge is at least 1/20

So in practical terms a carrier deck of thickness 12 inches will still fall into this category

However a 1/8th of an inch thick triangular web which is 1 inch on all sides inside an electronics module chassis is not a thin shell. It will have greater transverse shear stiffness

The deformations of the plate should be small compared to the plate thickness.

In the deck example, deflections of order 5 or 6 inches are very reasonable

Deflections of the order 50 inches would be suspect

In this case the concern would be that pure bending response is being supplemented by transition to induced membrane loads due to the large displacement.

This would mean a nonlinear large displacement analysis would be required, with accompanying changes in plate stiffness.

For composite type structures, the shear deformations through the thickness of the laminate can be important.

The shear stiffness of a sandwich panel with aluminum facing sheets and aramid honeycomb core, for example, is much lower than a homogeneous aluminum billet.

This means the shear stiffness terms in thin shells composites cannot be ignored.

Most composite shell elements cannot represent transverse shear stress well and ignore through thickness stresses

To overcome this limitation:

Shear deformation theories are combined with classical laminate theory.

The two systems are assumed to be superimposed.

There are various examples of shear deformation theory, ranging from basic linear assumption to high order series solutions.

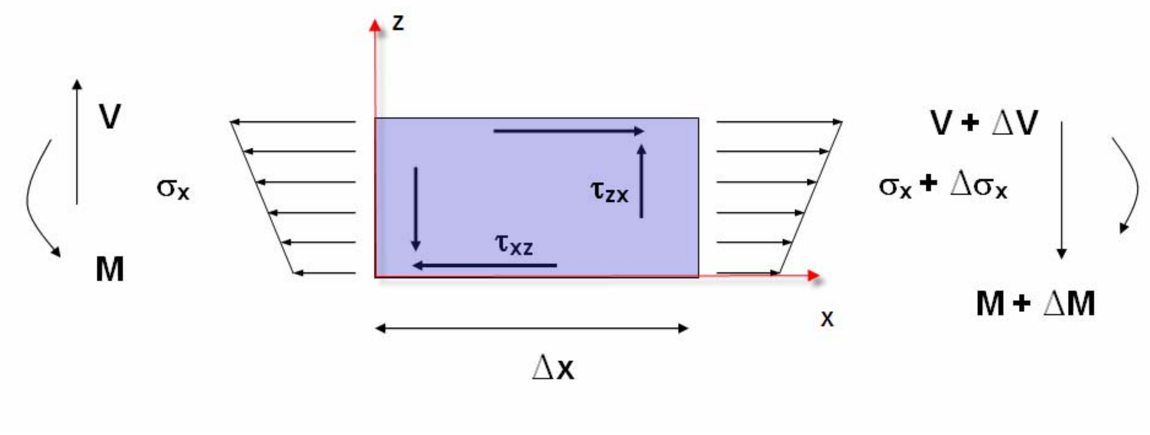
A popular method in NASTRAN solutions, for example, is to assume an equivalent mean effective transverse shear modulus to exist.

The in-plane stresses due to bending of the plate about each of the orthogonal radii of curvature are assumed to be independent.

This allows each sectional slice of laminate to be treated as a simple beam.

Using beam theory, the axial equilibrium and moment equilibrium conditions at a finite length of slice are used to derive balancing transverse shear stresses on a ply by ply basis.

So the ‘missing’ transverse shear $\partial\tau_{xz}$ is accounted for



Axial equilibrium
$$\frac{\partial \tau_{xz}}{\partial z} + \frac{\partial c_{x}}{\partial x} = 0$$

Vertical equilibrium
$$V_x + \frac{\partial M_x}{\partial x} = 0$$

There are two significant implications here

This is really only a bending response

The effect of any external applied force over length dx is ignored

The distribution of shear through depth follows a classical distribution across a rectangular section with 5/6 effective shear area

When does this matter?

For many thin shell type applications the effect can be ignored

- Overall composite wing and fuselage stress and stiffness
- Overall composite hull, frame and deck stress and stiffness

For Local Detail design can be very important

- Jointed regions – bonded or mechanical
- T connections, splice connections
- Edge regions

Use of 3D Solid Elements

A typical 3D composite element, such as that used in NEiNastran, does not have these limitations on transverse shear

The transverse shear and also the through thickness normal stresses are included as a direct result of the triaxial stress state in the fully 3D orthotropic material.

Downside – meshing complexity, speed of analysis

Use of 3D Solid Elements

Full 3d stress field

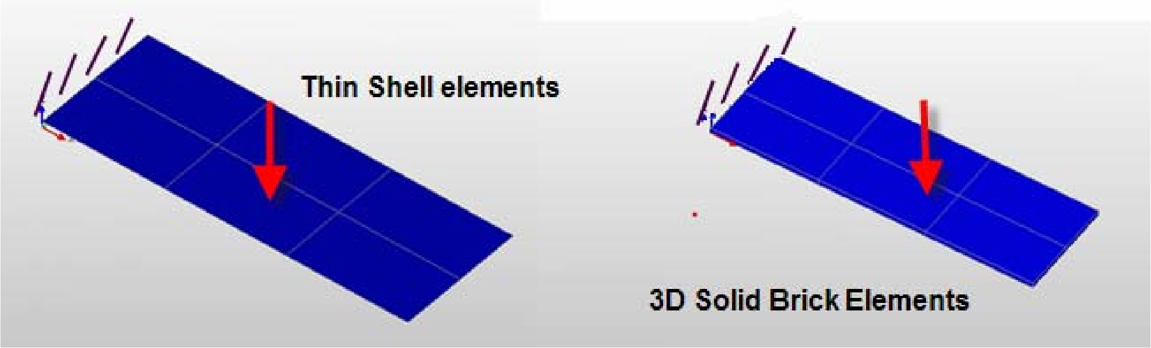
- Default 3 integration points per ply (extendable)
- Through thickness stresses
- Inter lamina shear stresses
- Zero shears at top/bottom surface

- HEX and PENTA elements
- Mapping of 3D ply layup from existing 2D PCOMP data
- Conversion of MAT8 (2d orthotropic) to new MAT12 (3d orthotropic)

An example is now shown of both a 2D shell model and a corresponding 3D solid model.

Both are identical in geometry, ply thickness and ply layup.

The 3D orthotropic material of the solid has been reduced to an effective 2D orthotropic material to allow comparative results.



Both laminates are balanced and symmetric

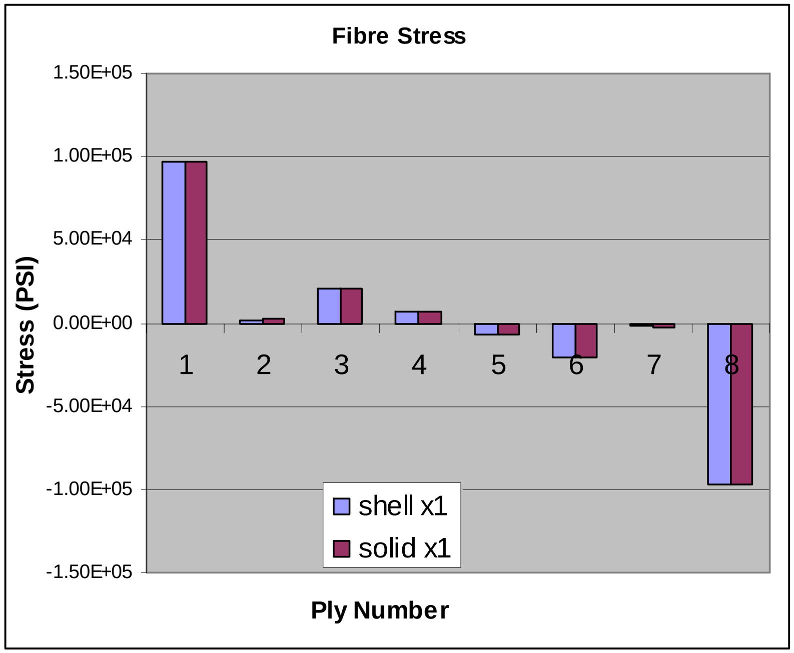
0/90/+45/-45/symm layups

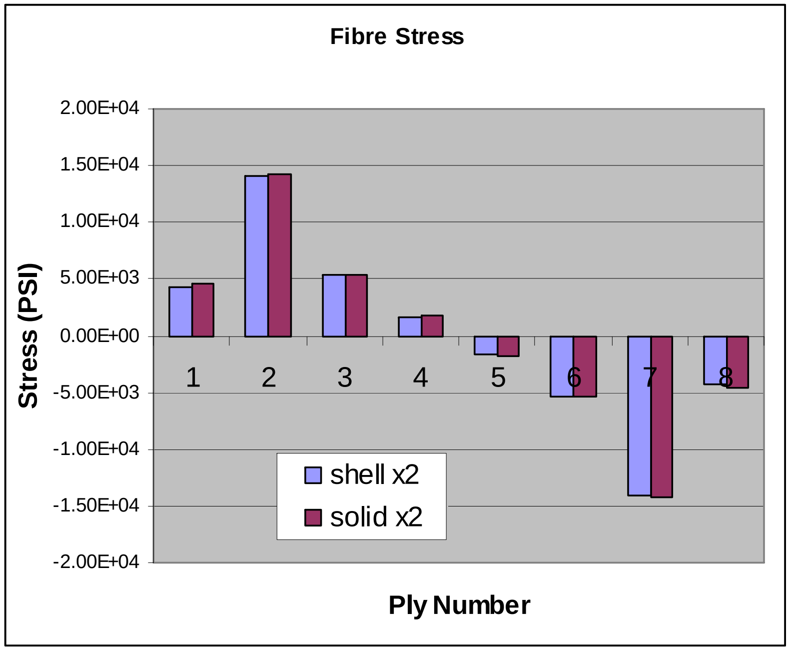
The loading was a distributed pressure over the face of the elements, with one end fully constrained.

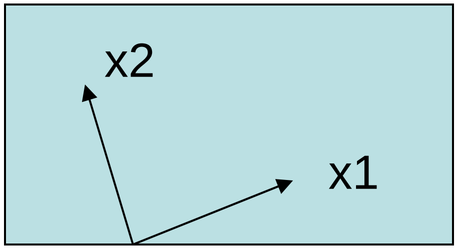
The ply stresses at the root element in both cases are considered, and the results are shown in the following figures.

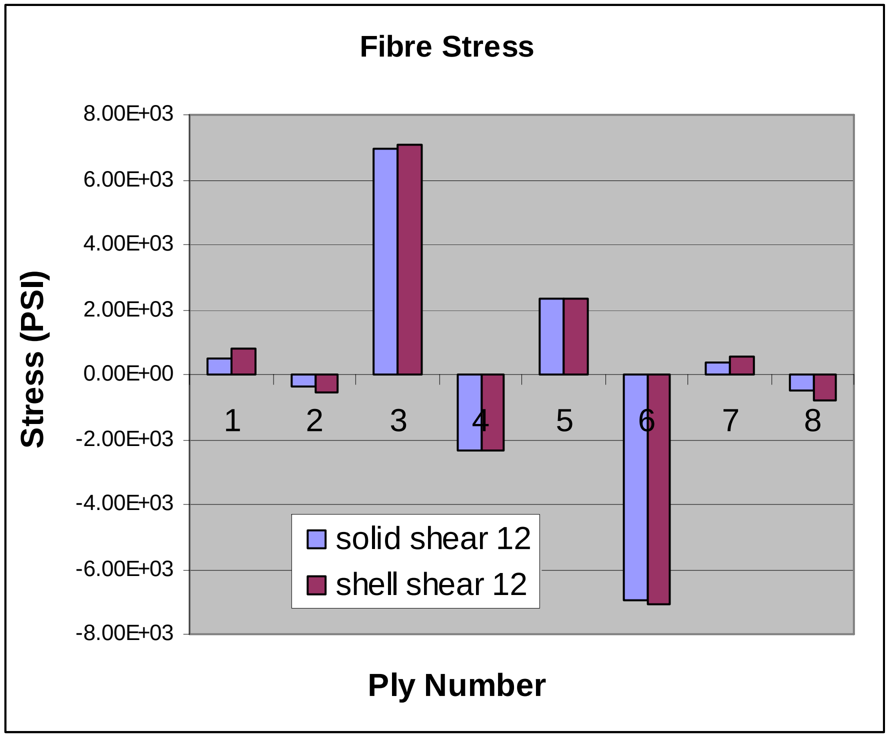
The results in the following slides show that very good agreement is achieved between the in-plane stresses throughout the laminate.

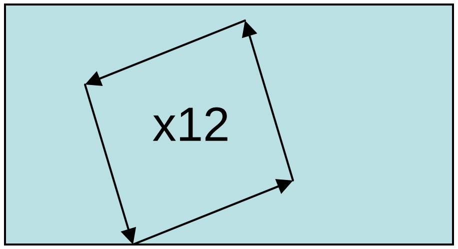
Classical laminate theory matches well with the directly derived stresses in the 3D elements.


Fibre Stress
1.50E+05
1.00E+05
5.00E+04
0.00E+00
-5.00E+04
-1.00E+05
-1.50E+05
Stress (PSI)
1
2
3
4
5
6
7
8
shell x1
solid x1
Ply Number


Fibre Stress
2.00E+04
1.50E+04
1.00E+04
5.00E+03
0.00E+00
-5.00E+03
-1.00E+04
-1.50E+04
-2.00E+04
Stress (PSI)
1
2
3
4
5
6
7
8
shell x2
solid x2
Ply Number


x2
x1

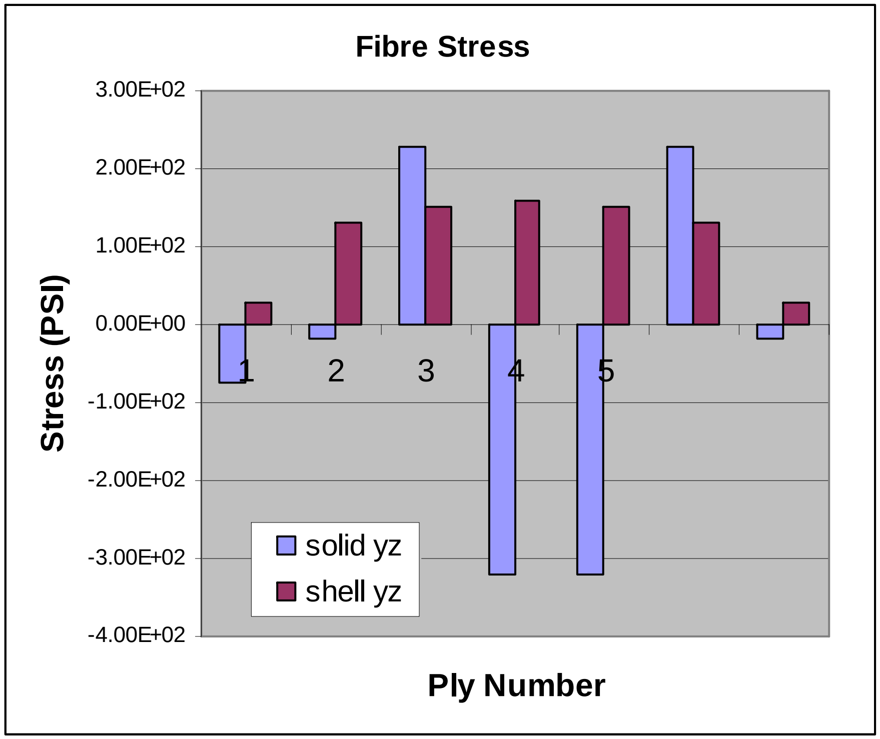

Fibre Stress
8.00E+03
6.00E+03
4.00E+03
2.00E+03
0.00E+00
-2.00E+03
-4.00E+03
-6.00E+03
-8.00E+03
Stress (PSI)
1
2
3
4
5
6
7
8
solid shear 12
shell shear 12
Ply Number

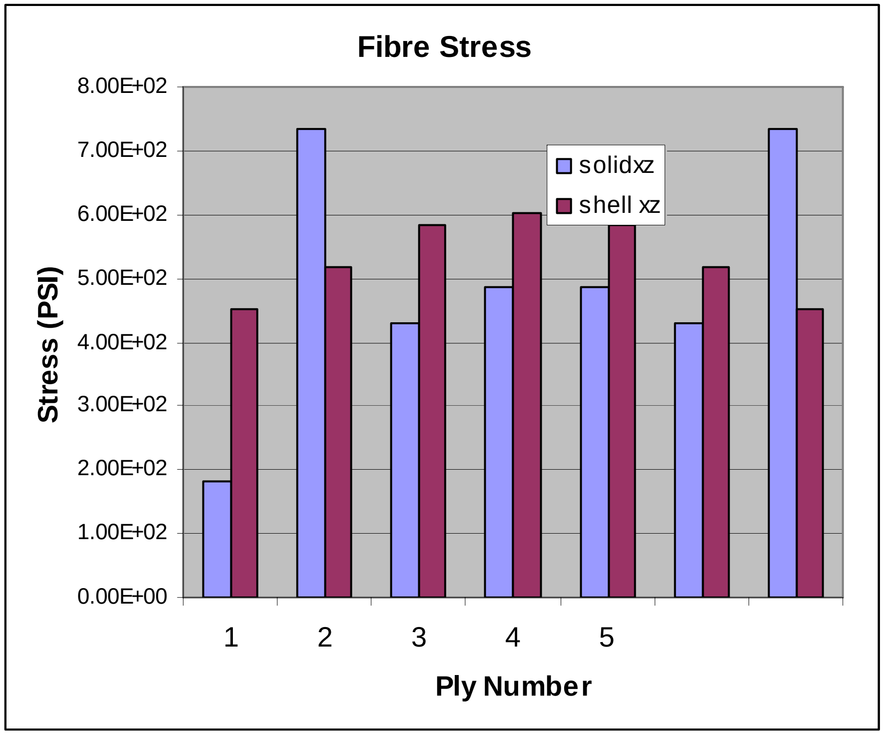

x12

However, the transverse shear stress results in xz and yz are very different.

The shell elements show a distribution that follows the parabolic shape of the simple beam shear theory.

The solid elements show a different distribution, although a similar magnitude.


Fibre Stress
3.00E+02
2.00E+02
1.00E+02
0.00E+00
-1.00E+02
-2.00E+02
-3.00E+02
-4.00E+02
Stress (PSI)
1
2
3
4
5
solid yz
shell yz
Ply Number


Fibre Stress
8.00E+02
7.00E+02
6.00E+02
5.00E+02
4.00E+02
3.00E+02
2.00E+02
1.00E+02
0.00E+00
Stress (PSI)
solidxz
shell xz
1
2
3
4
5
Ply Number

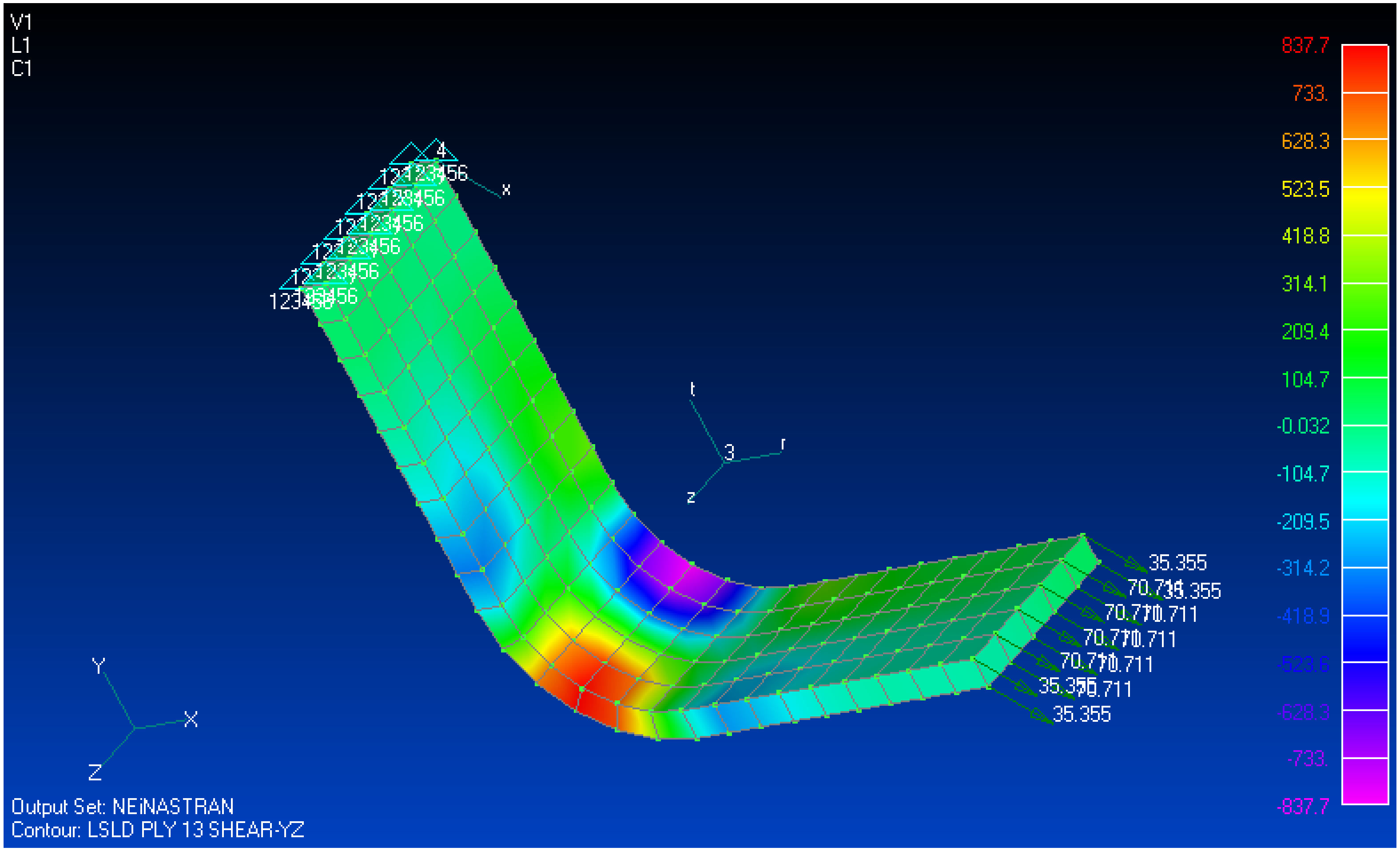
In this loading case, a normal pressure distribution, the 3D solids allow a through thickness direct stress to develop.

This balances the applied pressure loading.

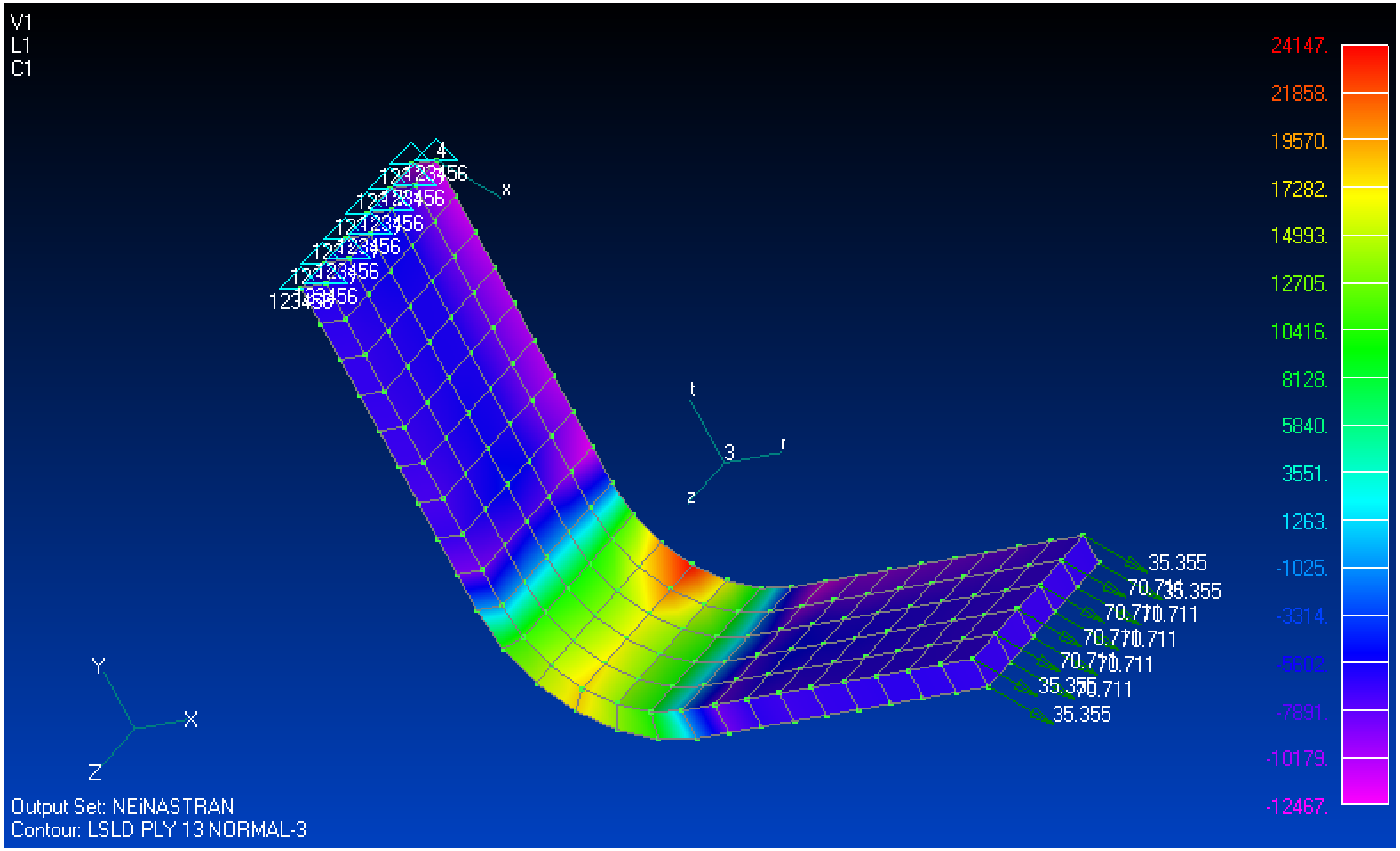
A complementary transverse shear state must exist to put this stress in equilibrium.

It is this stress state that is missed in the thin shell representation.

A curved composite panel is shown next, clearly indicating areas where through thickness and interlaminar stresses dominate



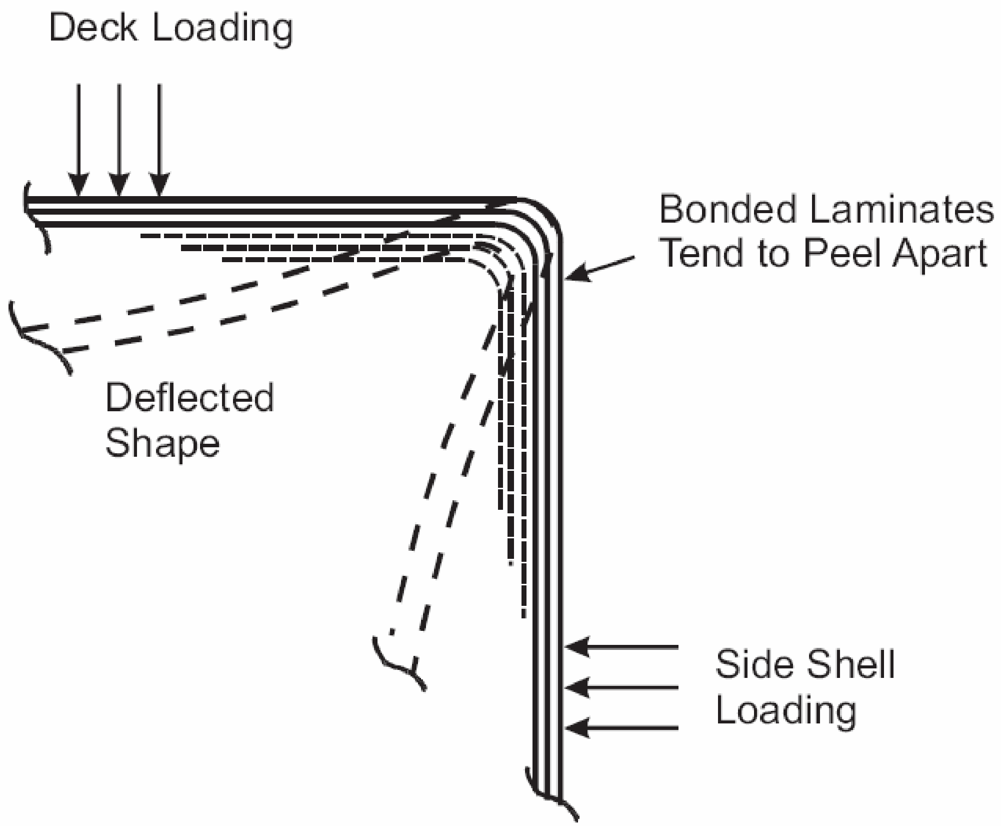
Output Set: NEiNASTRAN
Contour: LSLD PLY 13 SHEAR-YZ



Output Set: NEiNASTRAN
Contour: LSLD PLY 13 NORMAL-3

Interlaminar shear stresses are usually important in regions where an external boundary condition destroys the assumptions of classical laminate theory. The usual case considered is at the free edges of a component, but it has been shown here loading actions can also be important and may motivate the need to move away from shell elements.

Other examples are shown of structural geometry inducing through thickness stresses



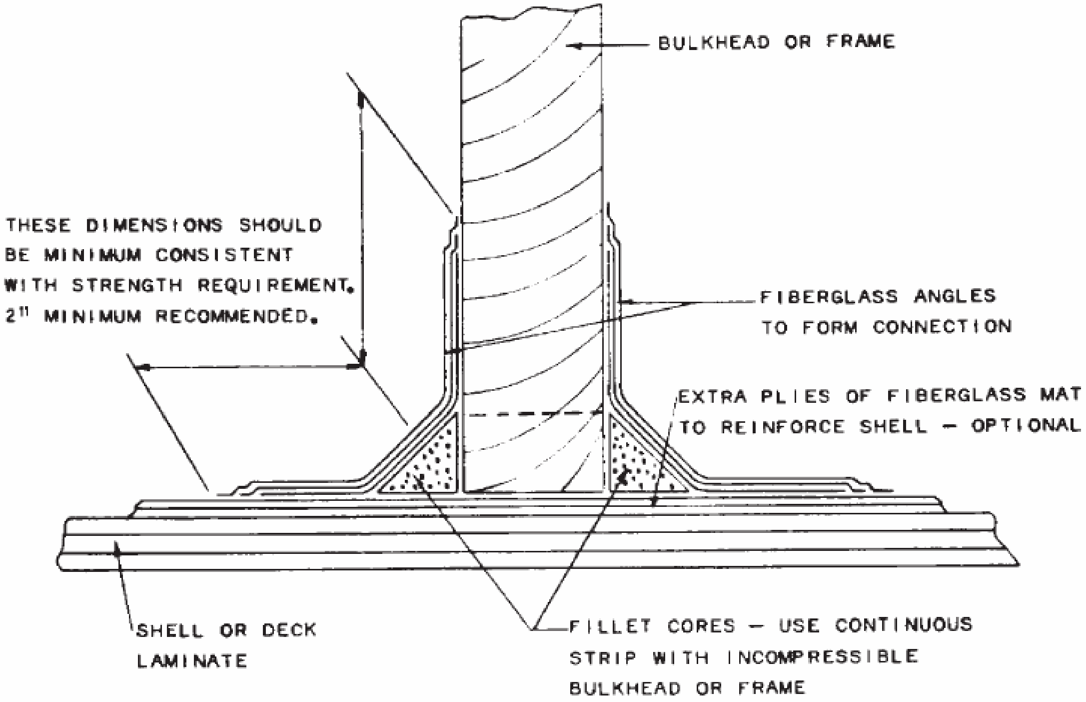
Deck Edge Connection - Normal Deck and Shell Loading Produces Tension at the Joint [Gibbs and Cox, ***Marine Design Manual for FRP***]

Plane Strain Assumptions

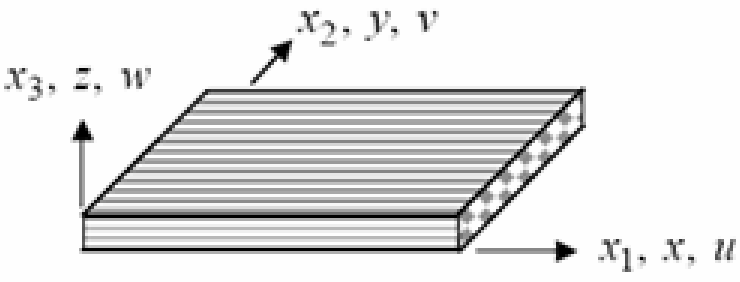
This idealization is typically used for cross sections of monolithic prismatic structures such as earthworks features, heavy bridge girders.

The motivation is to get detailed local stress information without having to carry out large scale 3D modeling.

At first sight there are distinct contradictions with these assumptions and recognized composite behaviour. However, if the structure is effectively infinite in the out of plane direction, and balanced sufficiently so that warping out of the plane is negligible, then a reasonable analysis can be made.



Connection of Bulkheads and Framing to Shell or Deck [Gibbs and Cox, ***Marine Design Manual for FRP***]



Plane strain

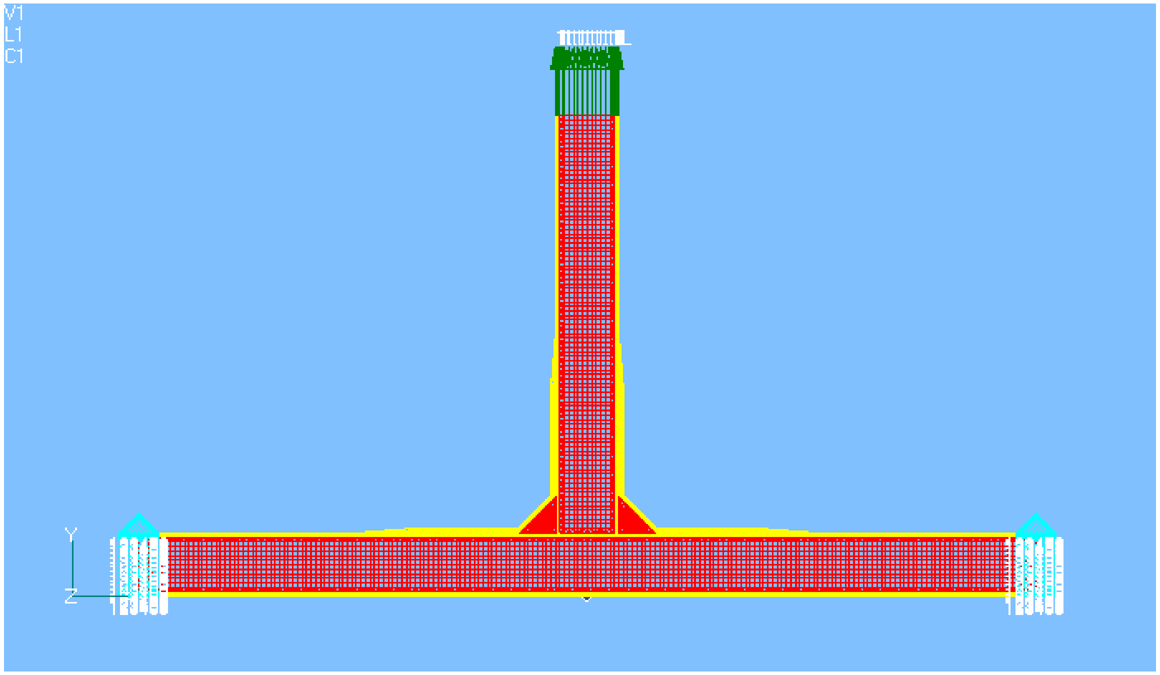
$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$$

$$\therefore \quad \tau_{xz} = \tau_{yz} = 0$$

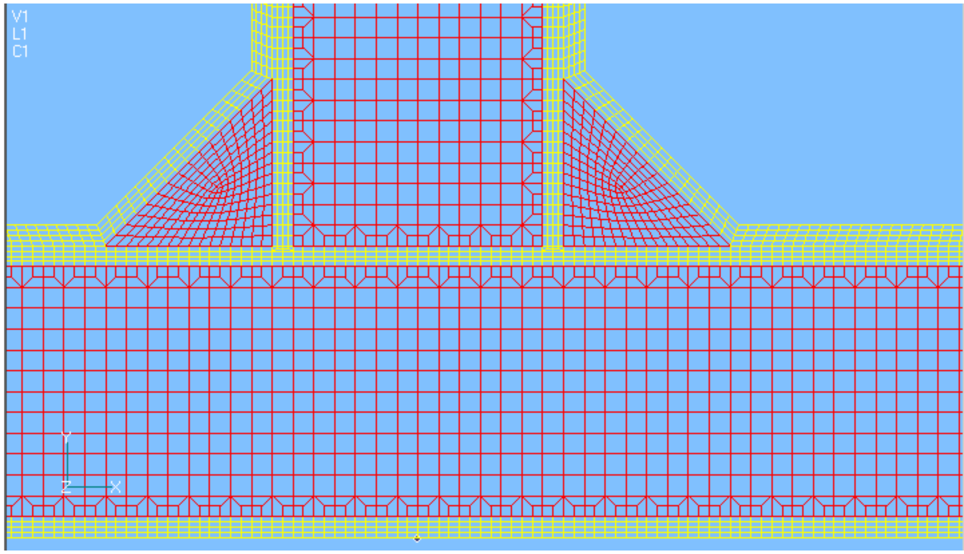
Plane Strain

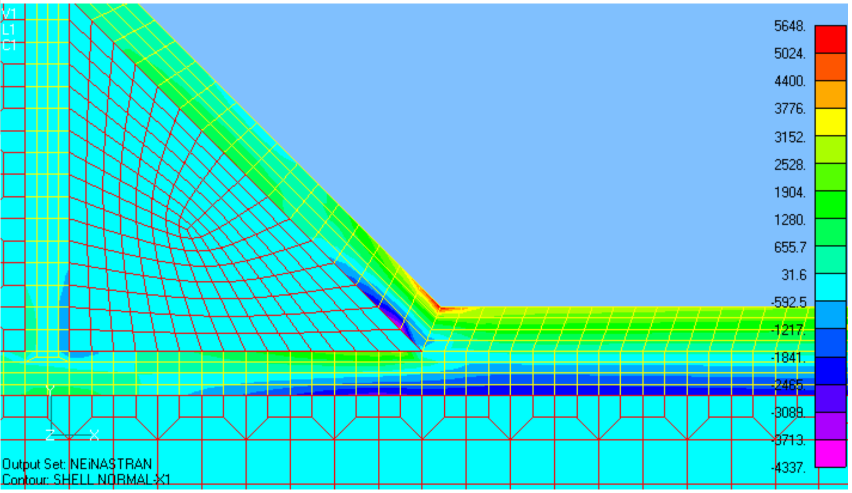
An example is shown of a model of a test specimen, used to investigate T type joints made of glass fibre containing balsa fillers bonded with adhesive layers.

The planar elements have been defined as plane strain elements, only in-plane terms can be defined.



Plane Strain





Plane Strain

Ex is the “smeared” equivalent orthotropic property for the entire laminate stacking sequence in the nominal ‘with fibre’ direction.

Ey is the “smeared” equivalent orthotropic property for the entire laminate stacking sequence in the nominal ‘transverse fibre’ direction.

Gxz, nuxz and nuxy are similiarly obtained.

The plane strain FEA axis set is 1 and 2 in-plane and 3 out of plane.

The following substitutions are made:

E1 = Ex
E2 = Ez
E3 = Ey – effectively ignored in basic plane strain analysis
G12 = Gxz
nu12 = nuxz
nu13 = nuxy - effectively ignored in basic plane strain analysis

Advanced Failure Criteria for Sandwich Structures

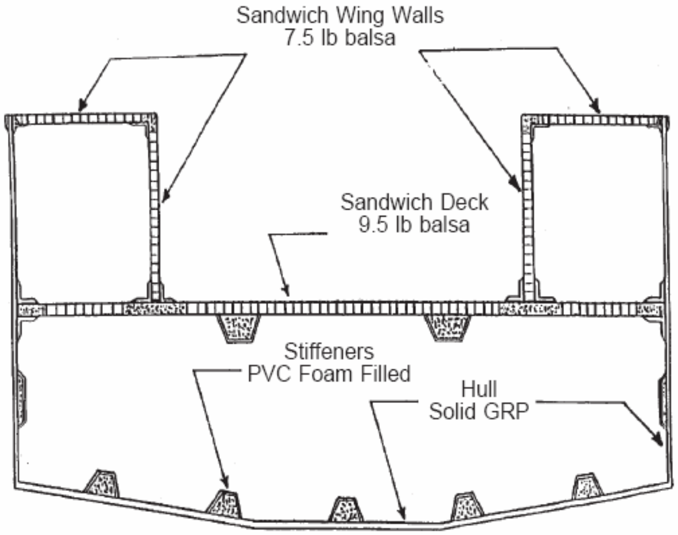


Figure 1-33 Lay-up Configuration for AMT Validation Model [Nguyen, *93 Sml Boat*]

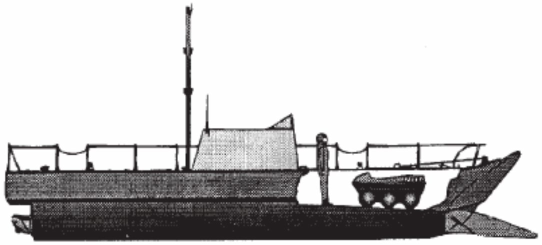


Figure 1-34 Profile of AMT Validation Model [Nguyen, *93 Sml Boat*]

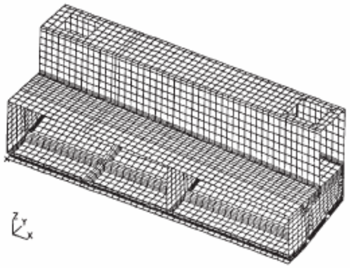


Figure 1-35 Midships FEM of AMT Validation Model [Nguyen, *93 Sml Boat*]

Advanced Failure Criteria for Sandwich Structures

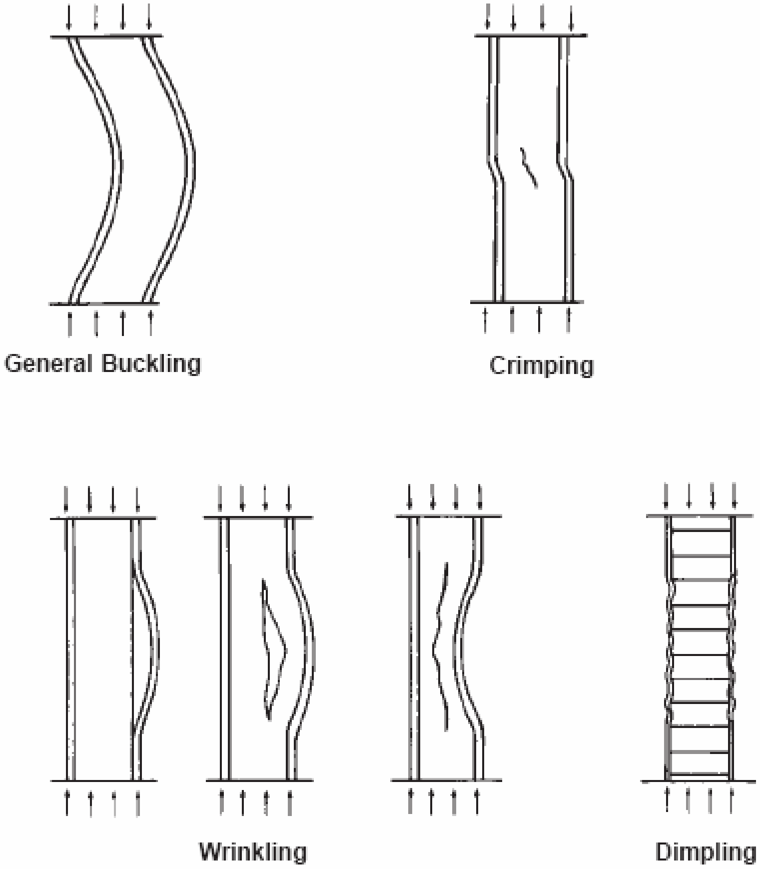
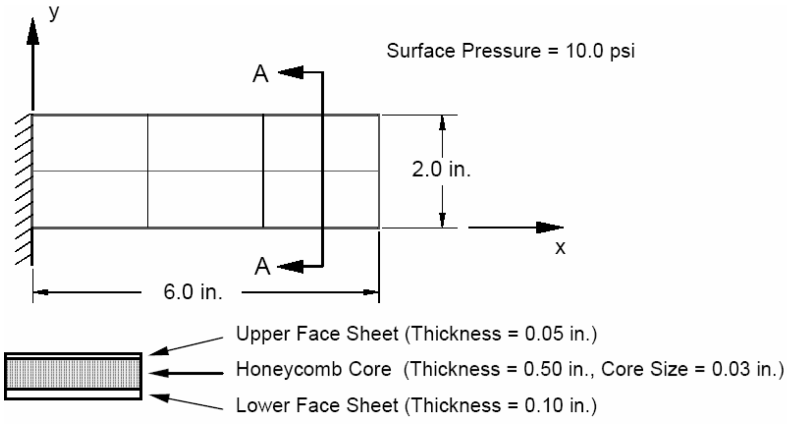


Figure 4-25 Compressive Failure Modes of Sandwich Laminates [*Sandwich Structures Handbook*, Il Prato]

Advanced Failure Criteria for Sandwich Structures



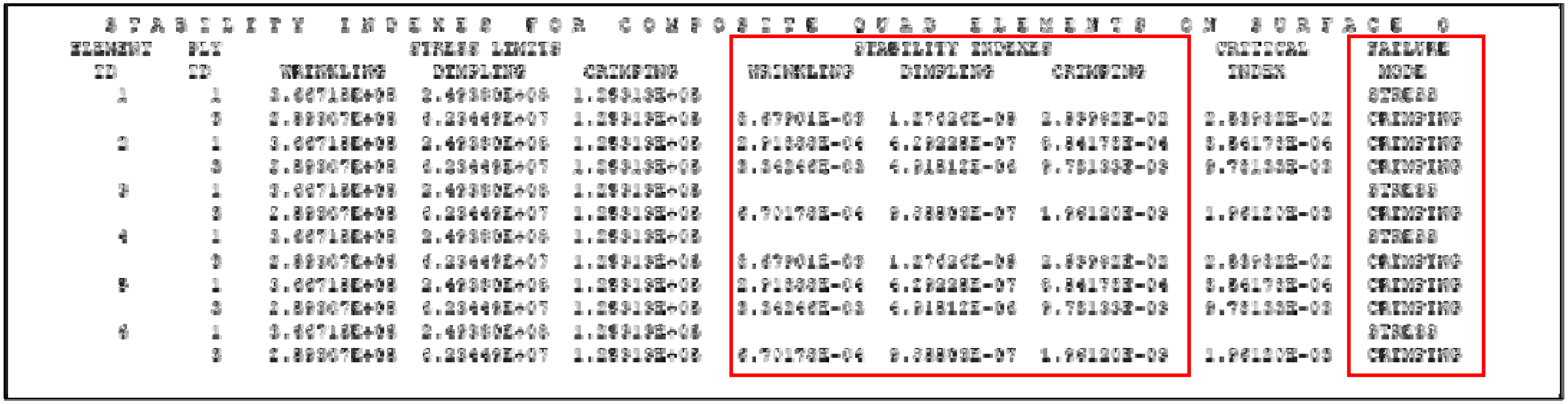
Surface Pressure = 10.0 psi

Upper Face Sheet (Thickness = 0.05 in.)

Honeycomb Core (Thickness = 0.50 in., Core Size = 0.03 in.)

Lower Face Sheet (Thickness = 0.10 in.)

Honeycomb Sandwich Results Output



```
         S T A B I L I T Y   I N D E X E S   F O R   C O M P O S I T E   Q U A D   E L E M E N T S   O N   S U R F A C E   0
   ELEMENT    PLY                STRESS LIMITS                                STABILITY INDEXES                   CRITICAL      FAILURE
     ID       ID       WRINKLING      DIMPLING       CRIMPING       WRINKLING      DIMPLING       CRIMPING         INDEX         MODE
      1        1      3.66713E+08    2.49380E+08    1.28313E+08                                                               STRESS
               3      2.89307E+08    6.23449E+07    1.28313E+08    8.67901E-03    1.27626E-08    2.35932E-02    2.35932E-02    CRIMPING
      2        1      3.66713E+08    2.49380E+08    1.28313E+08    2.71383E-04    4.29228E-07    5.84173E-04    5.84173E-04    CRIMPING
               3      2.89307E+08    6.23449E+07    1.28313E+08    3.34246E-03    4.91812E-06    9.78133E-03    9.78133E-03    CRIMPING
      3        1      3.66713E+08    2.49380E+08    1.28313E+08                                                               STRESS
               3      2.89307E+08    6.23449E+07    1.28313E+08    6.70178E-04    9.38808E-07    1.96120E-03    1.96120E-03    CRIMPING
      4        1      3.66713E+08    2.49380E+08    1.28313E+08                                                               STRESS
               3      2.89307E+08    6.23449E+07    1.28313E+08    8.67901E-03    1.27626E-08    2.35932E-02    2.35932E-02    CRIMPING
      5        1      3.66713E+08    2.49380E+08    1.28313E+08    2.71383E-04    4.29228E-07    5.84173E-04    5.84173E-04    CRIMPING
               3      2.89307E+08    6.23449E+07    1.28313E+08    3.34246E-03    4.91812E-06    9.78133E-03    9.78133E-03    CRIMPING
      6        1      3.66713E+08    2.49380E+08    1.28313E+08                                                               STRESS
               3      2.89307E+08    6.23449E+07    1.28313E+08    6.70178E-04    9.38808E-07    1.96120E-03    1.96120E-03    CRIMPING
```

Theory based on *Facesheet Wrinkling in Sandwich Structures*, NASA-CR-1999-208994, 1999

FEA Process for Composite Structure Analysis

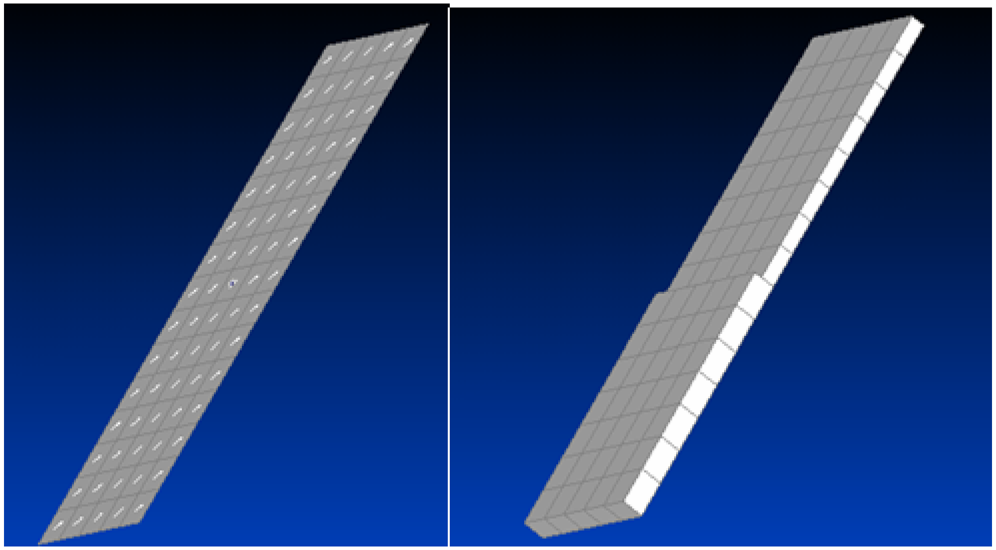
Some modelling strategies:

Global Model

- Thin shell 2d orthotropic elements model lay-ups and/or honeycomb face skins and core
- Solid isotropic model honeycomb core
- Solid 3D orthotropic elements model thick or tapering layups, orthotropic cores

Local Model

- 2D plane strain orthotropic elements
- Solid 3D orthotropic elements – ply by ply



planar view

3D view

- Account for Outer/Inner Mold Line continuity

Final Thoughts

- Actual material strength and stiffnesses are subject to many factors
- Fatigue, environmental degradation or abuse loads may dominate
- Published data – or generic test data, can only be a guide
- ‘As Built’ versus ‘As Designed’ in Composites can be significant
- FEA is not exact

Conclusions

- Aim for robust design (maximum xx microstrains everywhere)
- Keep engineering judgment in the picture
- Ignore progressive failure and fancy theories in initial design margins
- Test, analyze, ‘correlate’, etc. but test should be the final decider